7/12/2019

UNIT-4

Co-Ordinate Geometry

Formulas

* x-axis then $y=0$
* y-axis then $x=0$
* If x-axis is of the form $(x,0)$.
* If y-axis is of the form $(0,y)$
* Distance between two points $A=(x_1,y_1)$, $B=(x_2,y_2)$

Then

Distance

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

* A, B, C are collinear then $AB+BC=AC$

1. Find the distance between two points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

Sol Given

Two points

$$A=(\overset{x_1}{at_1^2}, \overset{y_1}{2at_1}),\ B=(\overset{x_2}{at_2^2}, \overset{y_2}{2at_2})$$

To find distance

We know that

Distance $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(at_2^2-at_1^2)^2+(2at_2-2at_1)^2}$$

$$= \sqrt{[a(t_2^2-t_1^2)]^2+[2a(t_2-t_1)]^2}$$

$$= \sqrt{a^2(t_2^2-t_1^2)^2+4a^2(t_2-t_1)^2}$$

$$= \sqrt{a^2[(t_2^2-t_1^2)^2+4(t_2-t_1)^2]}$$

$$= a\sqrt{(t_2^2-t_1^2)^2+4(t_2-t_1)^2}$$ //

H.W

2. Find the distance between two points (2,3) and (1,1)

Sol Given

Two points

$$A = \overset{x_1\ y_1}{(2,3)}, \quad B = \overset{x_2\ y_2}{(1,1)}$$

To find distance

We know that

$$\text{Distance } AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(1-2)^2+(1-3)^2}$$

$$= \sqrt{(-1)^2+(-2)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5}$$ //

3. Show that three points (0,−2), (−1,1), (−2,4) are collinear

Sol Given

Three points $A=(0,-2)$, $B(-1,1)$, $C(-2,4)$

To show that three points are collinear.

We know that

$$AB+BC = AC \longrightarrow (1)$$

By using distance formula

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-1-0)^2+(1+2)^2}$$

$$= \sqrt{1+(3)^2}$$

$$= \sqrt{1+9} = \sqrt{10}$$

$$BC = \sqrt{(-2+1)^2+(4-1)^2}$$

$$= \sqrt{1+9} = \sqrt{10}$$

$$AC = \sqrt{(2-0)^2+(4+2)^2}$$

$$= \sqrt{4+36} = \sqrt{40}$$

$$= 2\sqrt{10}$$

From eq ① becomes

$$\sqrt{10}+\sqrt{10} = 2\sqrt{10}$$

$$2\sqrt{10} = 2\sqrt{10}$$

$\therefore$ A, B, C are three points of collinear.

Types of triangles

1. Equilateral triangle $AB = BC = CA$
2. Isosceles triangle $\longrightarrow$ ①

   $AB = BC$ (or) $BC = CA$ (or) $CA = AB$
3. Right angle triangle $\rightarrow$ ②

   $AB^2 + BC^2 = CA^2$ (or) $AC^2 + AB^2 = BC^2$ (or) $AC^2 + BC^2 = AB^2$
4. Right angled Isosceles triangle

   From eq ① & ② satisfied.
5. Acute angle $0 < \theta < 90°$
6. Obtuse angle $90 < \theta < 180$

H.W

1. To show that $(2,4)$, $(2,6)$, $(2+\sqrt{3}, 5)$ form a equilateral triangle.

Sol Given

$A = (2,4)$, $B = (2,6)$, $C = (2+\sqrt{3}, 5)$

To show that A, B, C forms a equilateral triangle.

We know that

$$AB = BC = CA \longrightarrow ①$$

By using distance formula

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2-2)^2+(6-4)^2}$$

$$= \sqrt{(0)^2+(2)^2}$$

$$= \sqrt{4} = 2$$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2+\sqrt{3}-2)^2+(5-6)^2}$$

$$= \sqrt{\cancel{4}+3\cancel{-4}+(-1)^2}$$

$$= \sqrt{3+1}$$

$$= \sqrt{4} = 2$$

$$CA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2-2+\sqrt{3})^2+(4-5)^2}$$

$$= \sqrt{\cancel{4}-\cancel{4}+3+(-1)^2}$$

$$= \sqrt{3+1}$$

$$= \sqrt{4} = 2$$

From eq ① becomes

$AB = BC = CA$

$2 = 2 = 2$

$\therefore$ A, B, C are points forms a equilateral triangle

2. Show that $(4,4)$, $(3,5)$, $(-1,-1)$ form a right angle triangle

Sol Given $A = (4,4)$, $B = (3,5)$, $C = (-1,-1)$

To show that A, B, C form a right angle triangle

We know that

$$AC^2 + AB^2 = BC^2 \quad \text{—— ①}$$

By using distance formula

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-1-4)^2+(-1-4)^2}$$

$$= \sqrt{(-5)^2+(-5)^2}$$

$$= \sqrt{25+25}$$

$$= \sqrt{50} \quad (\because \sqrt{25\times 2} = 5\sqrt{2})$$

Squaring on both sides

$$AC^2 = (5)^2(\sqrt{2})^2$$

$$= 25\times 2 = 50$$

$$AC^2 = 50$$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(3-4)^2+(5-4)^2}$$

$$= \sqrt{(-1)^2+(1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

Squaring on both sides

$$AB^2 = (\sqrt{2})^2 = 2$$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{\cancel{(3-(-4))^2}\,(-1-3)^2+(-1-5)^2}$$

$$= \sqrt{(-4)^2+(-6)^2}$$

$= \sqrt{16+36}$

$= \sqrt{52}$

squaring on both sides

$BC^2 = (\sqrt{52})^2$

$BC^2 = 52$

From eq ①' becomes

$AC^2 + AB^2 = BC^2$

$50 + 2 = 52$

$52 = 52.$

$\therefore$ A, B and c points forms a right angle triangle

3. Show that $(-2,5)$, $(3,-4)$, $(7,10)$ form a right angle isosceles triangle.

Sol Given

$A = (-2,5)$, $B = (3,-4)$, $C = (7,10)$

To show that A, B, C form a right angle isosceles triangle

We know that right angle isosceles triangle satisfies Isosceles triangle $AB = CA$

Right angle triangle $AB^2 + AC^2 = BC^2$

By using distance formula $AB^2 + CA^2 = AB^2$

$A = (\overset{x_1}{-2}, \overset{y_1}{5})$, $B = (\overset{x_2}{3}, \overset{y_2}{-4})$

$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(3-(-2))^2 + (-4-5)^2}$

$= \sqrt{(3+2)^2 + (-9)^2}$

$= \sqrt{(5)^2 + 81}$

$= \sqrt{25+81}$

$= \sqrt{106}$

Squaring on both sides

$AB^2 = 106$ //

$B = (3, -4)$, $C = (7, 10)$
$x_1 \; y_1$, $x_2 \; y_2$

$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-3)^2+(10-(-4))^2}$

$= \sqrt{(4)^2+(10+4)^2}$

$= \sqrt{16+196}$

$= \sqrt{212}$

Squaring on both sides

$BC^2 = 212$ //

$C = (7, 10)$, $A = (-2, 5)$
$x_1 \; y_1$, $x_2 \; y_2$

$CA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-7)^2+(5-10)^2}$

$= \sqrt{(-9)^2+(-5)^2}$

$= \sqrt{81+25}$

$= \sqrt{106}$

Squaring on both sides

$CA^2 = 106$ //

It satisfies Isosceles triangle

$AB = CA$

$106 = 106$

It satisfies Right angle triangle

$AB^2 + AC^2 = CB^2$

$106 + 106 = 212$

$212 = 212$ //

10/12/2019 Area of the triangle

Given (x_1, y_1), (x_2, y_2), (x_3, y_3) are three points then area of the triangle $= \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$

* If area of the triangle is zero then the points are collinear

1. Find the area of the triangle by the three points $(1,2)$, $(3,-4)$, $(-2,0)$

Sol. Given points $A=(\underset{x_1}{1},\underset{y_1}{2})$, $B=(\underset{x_2}{3},\underset{y_2}{-4})$; $C=(\underset{x_3}{-2},\underset{y_3}{0})$

To find area of the triangle

We know that

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1-3 & 1-(-2) \\ 2-(-4) & 2-0 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 3 \\ 6 & 2 \end{vmatrix}$$

det = ad − bc

$$= \frac{1}{2}\left|-4-18\right|$$

$$= \frac{1}{2}\left|-22\right|$$

$$= \frac{1}{\cancel{2}}\left|\overset{11}{\cancel{22}}\right|$$

$= 11$ sq. units

2. If the area of the triangle formed by $(1,2)$, $(2,3)$, $(x,4)$ is 40 sq. units then find x.

Sol. Given points $A=(\underset{x_1}{1},\underset{y_1}{2})$, $B=(\underset{x_2}{2},\underset{y_2}{3})$, $C=(\underset{x_3}{x},\underset{y_3}{4})$

and also given area of the triangle is 40 sq. units

We know that

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$$

$$40 = \frac{1}{2}\begin{vmatrix} 1-2 & 1-x \\ 2-3 & 2-4 \end{vmatrix}$$

$$40 \times 2 = \begin{vmatrix} -1 & 1-x \\ -1 & -2 \end{vmatrix} \qquad [ad-bc]$$

$$80 = |-1(1-x)-(2)|$$

$$80 = |-1+x-2|$$

$$80 = |-3+x|$$

$$80 = 3+x$$

$$80-3 = x$$

$$\therefore \cancel{x = 77}$$

$$80 = |2-(1-x)(-1)|$$

$$80 = |2-(-1+x)|$$

$$80 = |2+1-x|$$

$$80 = 3-x$$

$$80-3 = -x$$

$$77 = -x$$

$$\therefore \boxed{x = -77}$$

3. Find the values of 't' if the points $(t, 2t)$, $(2t, 6t)$, and $(3, 8)$ are collinear.

Sol Given points $\underset{x_1\ y_1}{(t, 2t)}$, $\underset{x_2\ y_2}{(2t, 6t)}$, $\underset{x_3\ y_3}{(3, 8)}$ are collinear

We know that

Area of triangle is zero

To find 't'

$$\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} t-2t & t-3 \\ 2t-6t & 2t-8 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} -t & t-3 \\ -4t & 2t-8 \end{vmatrix} = 0$$

$$= \frac{1}{2}|-t(2t-8)-(-4t)(t-3)| = 0$$

$$= \frac{1}{2}|-2t^2+8t+4t^2-12t| = 0$$

$$= |2t^2-4t| = 0 \times 2$$

$$= 2t(t-2) = 0$$

$$2t = 0 \qquad t-2 = 0$$

$$\boxed{t = 0} \qquad \boxed{t = 2}$$

//

4. If $(k, 2-2k)$, $(-k+1, 2k)$, $(-4-k, 6-2k)$ are collinear then find 'k' $8k^2+4k-4=0$ $k=-1, k=\frac{1}{2}$

Sol: Given points

$$\underset{x_1\ \ y_1}{(k, 2-2k)}, \underset{x_2\ \ y_2}{(-k+1, 2k)}, \underset{x_3\ \ y_3}{(-4-k, 6-2k)}$$

Area of triangle is zero

To find 'k'

We know that

$$\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} k-(-k+1) & k-(-4-k) \\ 2-2k-2k & 2-2k-6+2k \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2k-1 & 2k+4 \\ 2-4k & -4 \end{vmatrix} = 0$$

$$\Rightarrow (2k-1)\times -4 - 2-4k\,(2k+4) = 0$$

$$\Rightarrow -8k+4-(4k+8-8k^2-16k) = 0$$

$$\Rightarrow -8k+4-4k-8+8k^2+16k = 0$$

$$\Rightarrow 8k^2+4k-4 = 0$$

$$\Rightarrow 8k^2+8k-4k-4 = 0$$

$$\Rightarrow 8k(k+1)-4(k+1) = 0$$

$$\Rightarrow (k+1)(8k-4) = 0$$

$k+1=0$, $\boxed{k=-1}$ | $8k-4=0$, $8k=4$, $k=\cancel{4}/\cancel{8}\,2$, $\boxed{k=\frac{1}{2}}$

Find the value of 'k', if the distance between two points $(1,0),(4,k)$ is 5

Sol Given $A=(1,0)$ [x_1 y_1], $B=(4,k)$ [x_2 y_2]

and distance $=5$

To find 'k'

We know that

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$5 = \sqrt{(4-1)^2+(k-0)^2}$$

$$5 = \sqrt{(3)^2+(k)^2}$$

$$5 = \sqrt{9+k^2}$$

$$5 = \sqrt{9+k^2}$$

Squaring on both sides

$$25 = \left(\sqrt{9+k^2}\right)^2$$

$$25 = 9+k^2$$

$$25-9 = k^2$$

$$16 = k^2$$

$$\therefore k = \sqrt{16}$$

$$\boxed{k=4}$$

11/12/2019

Section formula:

* The point 'p' which divides the line segment joining the points $A(x_1,y_1)$, $B(x_2,y_2)$ in the ratio $m:n$

internally $\left[\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right]$

Externally $\left[\dfrac{mx_2-nx_1}{m-n}, \dfrac{my_2-ny_1}{m-n}\right]$

Find the point which divides the line segment joining (-1, 2) and (4, -5) in the ratio 3:2

Sol Given points

$$(\underset{x_1}{-1}, \underset{y_1}{2}), (\underset{x_2}{4}, \underset{y_2}{-5})$$

and ratio $\underset{m}{3} : \underset{n}{2}$

We know that

$$\text{Internally} = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$$

$$= \left[\frac{3\times4+2\times(-1)}{3+2}, \frac{3\times(-5)+2(2)}{3+2}\right]$$

$$= \left[\frac{12-2}{5}, \frac{-15+4}{5}\right]$$

$$= \left[\frac{\cancel{10}\,2}{\cancel{5}}, \frac{-11}{5}\right]$$

$$= \left[2, -\frac{11}{5}\right]$$

Trisection point

The ratio 1:2 and 2:1 is called trisection point..

1. Find the point of trisection of line segment joining the points (1,3), (4,2)

Sol Step-1

Given points $(\underset{x_1}{1}, \underset{y_1}{3}), (\underset{x_2}{4}, \underset{y_2}{2})$ and $\underset{m}{1} : \underset{n}{2}$

We know that

$$\text{Internally} = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$$

$$= \left[\frac{1\times4+2\times1}{1+2}, \frac{1\times2+2\times3}{1+2}\right]$$

$$= \left[\frac{4+2}{3}, \frac{2+6}{3}\right]$$

$$= \left[\frac{\cancel{6}\,2}{\cancel{3}}, \frac{8}{3}\right]$$

$$\left[2, \frac{8}{3}\right]$$

Step-2

Given points $(1,3)$, $(4,2)$ and $2:1$ ratio
x_1, y_1 ; x_2, y_2 ; m n

We know that

$$\text{Internally} = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$$

$$= \left[\frac{2\times4+1\times1}{2+1}, \frac{2\times2+1\times3}{2+1}\right]$$

$$= \left[\frac{8+1}{3}, \frac{4+3}{3}\right]$$

$$= \left[\frac{\cancel{9}\,3}{\cancel{3}}, \frac{7}{3}\right]$$

$$= \left[3, \frac{7}{3}\right]$$

* If $p(x,y)$ lines in the joining $A(x_1,y_1)$, $B(x_2,y_2)$, the 'p' divide AB in the ratio is $\boxed{x_1-x : x-x_2}$

1. Find the ratio in which $(1,4)$ divides the segment joining $(2,1)$, $(4,-5)$

Sol Given in which $(\overset{x}{1},\overset{y}{4})$ divides the segment joining $(\overset{x_1}{2},\overset{y_1}{1})$, $(\overset{x_2}{4},\overset{y_2}{-5})$

To find ratio

We know that

Ratio $\rightarrow$ $x_1-x : x-x_2$

$2-1 : 1-4$

$1 : -3$ ($\because$ ratio is always in +ve value).

$1:3$

* If x-axis divide the line segment joining points (x_1,y_1) (x_2,y_2) in the ratio is $\boxed{-y_1 : y_2}$

1. Find the ratio in which x-axis divides the segment joining $(1,-3)$, $(5,7)$

Sol Given

In which x-axis divides the segment joining

$(\underset{x_1}{1}, \underset{y_1}{-3})$, $(\underset{x_2}{5}, \underset{y_2}{7})$

To find ratio

We know that

Ratio $\Rightarrow$ $-y_1 : y_2$

$-(-3) : 7$

$3 : 7$

* If y-axis divide the line segment joining points (x_1, y_1) (x_2, y_2) in the ratio is $\boxed{-x_1 : x_2}$

1. Find the ratio in which y-axis divides the segment joining $(-2,4)$, $(6,-1)$.

Sol Given

In which y-axis divides the segment joining $(\underset{x_1}{-2}, \underset{y_1}{4})$ $(\underset{x_2}{6}, \underset{y_2}{-1})$

To find ratio

We know that

Ratio $\Rightarrow$ $-x_1 : x_2$

$-(-2) : 6$

$\cancel{2} : \cancel{6}\,3$

$1 : 3$ //

Centroid:

Given (x_1, y_1), (x_2, y_2), (x_3, y_3) then centroid

$$G = \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

1. Find the centroid of triangle formed by $(1,2)$, $(3,7)$, $(6,1)$

Sol Given points

$(1,2), (3,7), (6,1)$
$\;\;x_1\,y_1 \quad x_2\,y_2 \quad x_3\,y_3$

To find Centroid

We know that

$$\text{Centroid } G = \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

$$= \left[\frac{1+3+6}{3}, \frac{2+7+1}{3}\right]$$

$$= \left[\frac{10}{3}, \frac{10}{3}\right]$$

Question

* The centroid of triangle where vertices are $(2,4), (3,k)$ and $(4,2)$ is $(k,3)$. Find k.

Sol Given points $(2,4), (3,k), (4,2)$
$\;\;x_1\,y_1 \quad x_2\,y_2 \quad x_3\,y_3$

Centroid $G = (k,3)$

We know that

$$G = \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

$$(k,3) = \left[\frac{2+3+4}{3}, \frac{4+k+2}{3}\right]$$

$$(k,3) = \left[\frac{\cancel{9}3}{\cancel{3}}, \frac{6+k}{3}\right]$$

$$(k,3) = \left[3, \frac{6+k}{3}\right]$$

$3 = \frac{6+k}{3}$

$3 \times 3 = 6+k$

$9-6 = k$

$\therefore \boxed{k=3}$

$$\therefore \boxed{k=3}$$

If two vertices of triangle are $(4,8), (-2,6)$ and centroid is $(2,7)$ then find third vertex.

Sol Given two points $(\overset{x_1\,y_1}{4,8}), (\overset{x_2\,y_2}{-2,6})$

let third vertex (x_3, y_3)

and centroid $G = (2,7)$

We know that

$$G = \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

$$(2,7) = \left[\frac{4+(-2)+x_3}{3}, \frac{8+6+y_3}{3}\right]$$

$$(2,7) = \left[\frac{4-2+x_3}{3}, \frac{14+y_3}{3}\right]$$

$$(2,7) = \left[\frac{2+x_3}{3}, \frac{14+y_3}{3}\right]$$

$2 = \frac{2+x_3}{3}$	$7 = \frac{14+y_3}{3}$
$6-2 = x_3$	$21-14 = y_3$
$4 = x_3$	$7 = y_3$
$\therefore \boxed{x_3 = 4}$	$\therefore \boxed{y_3 = 7}$

$\therefore$ Third vertex $(x_3, y_3) = (4,7)$.

12/12/2019

Formulas

* Slope of x-axis is 0.
* slope of y-axis is $1/0$.
* Equation of x-axis is $y=0$
* Equation of y-axis is $x=0$
* Given (x_1, y_1), (x_2, y_2) then slope $m = \frac{y_2-y_1}{x_2-x_1}$

i. Find slope of line joining of points $(1,2)$, $(3,4)$

Sol. Given points $\overset{x_1\, y_1}{(1,2)}$, $\overset{x_2\, y_2}{(3,4)}$

To find slope

We know that

$$\text{Slope } m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{4-2}{3-1} = \frac{\cancel{2}}{\cancel{2}} = 1$$

* If θ is acute angle between lines
then $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

1. Find the acute angle between lines having slopes $1/7, -3/4$

Sol Given slopes $m_1 = \frac{1}{7}$, $m_2 = \frac{-3}{4}$

To find acute angle (θ)

We know that

$$\tan\theta = \boxed{\left|\frac{m_1-m_2}{1+m_1m_2}\right|}$$

$$\tan\theta = \left|\frac{1/7-(-3/4)}{1+(1/7)(-3/4)}\right| = \left|\frac{\frac{4+21}{28}}{1-3/28}\right|$$

$$\tan\theta = \left|\frac{\cancel{\frac{4+21}{28}}}{\cancel{\frac{28-3}{28}}}\right|$$

$$\tan\theta = \left|\frac{\cancel{25}}{\cancel{25}}\right|$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = 45^\circ}$$

Equation of the straight line in different form

Formula.

* Given point (x_1, y_1) and slope 'm' then equation of the straight line $\boxed{y-y_1 = m(x-x_1)}$

1. Find the equation of the straight line passing through the points $(1,2)$ having slopes $1/2$.

Sol Given point $\overset{x_1\,y_1}{(1,2)}$ and slope $m = 1/2$

To find equation of straight line

We know that

equation of straight line is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \tfrac{1}{2}(x-1)$$
$$2(y-2) = 1(x-1)$$
$$2y - 4 = x - 1$$
$$x - 1 - 2y + 4 = 0$$
$$x - 2y + 3 = 0$$

* Given two points (x_1, y_1) (x_2, y_2) then equation of straight line is $\boxed{\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}}$

1. Find the equation of the straight line passing through points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$

Sol Given two points $(\overset{x_1}{at_1^2}, \overset{y_1}{2at_1})$, $(\overset{x_2}{at_2^2}, \overset{y_2}{2at_2})$

To find equation of straight line.

We know that equation of the straight line is

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-2at_1}{x-at_1^2} = \frac{2at_2-2at_1}{at_2^2-at_1^2}$$

$$\Rightarrow \frac{y-2at_1}{x-at_1^2} = \frac{2\cancel{a}(t_2-t_1)}{\cancel{a}(t_2^2-t_1^2)} \quad \{a^2-b^2 = (a+b)(a-b)$$

$$\Rightarrow \frac{y-2at_1}{x-at_1^2} = \frac{2\cancel{(t_2-t_1)}}{\cancel{(t_2-t_1)}(t_2+t_1)}$$

$$\Rightarrow (t_2+t_1)(y-2at_1) = 2(x-at_1^2)$$

$$\Rightarrow y(t_2+t_1) - 2at_1(t_2+t_1) = 2x - 2at_1^2$$

$$\Rightarrow -y(t_2+t_1) + 2x - 2at_1^2 + 2at_1(t_2+t_1) = 0$$

$$\Rightarrow 2x - y(t_2+t_1) + 2at_1t_2 + \cancel{2at_1^2} - \cancel{2at_1^2} = 0$$

$$\Rightarrow 2x - y(t_2+t_1) + 2at_1t_2 = 0$$

(2.) Find the equation of the straight line passing through the points $(3,4)$, $(7,-6)$ Ans: $5x+2y-23=0$

Sol Given points $(\overset{x_1, y_1}{3,4})$, $(\overset{x_2, y_2}{7,-6})$

To find equation of straight line

We know that equation of the straight line is

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-4}{x-3} = \frac{-6-4}{7-3}$$

$$\frac{y-4}{x-3} = \frac{-10}{4}$$

$$4(y-4) = -10(x-3)$$

$$4y-16 = -10x+30$$

$$4y-16+10x-30 = 0$$

$$10x+4y-46 = 0$$

$$2(5x+2y-23) = 0$$

$$5x+2y-23 = 0/2$$

$$5x+2y-23 = 0 \,//$$

$$4(y-4) = -10(x-3)$$
$$4y-16 = -10x+30$$
$$4y-16+10x-30 = 0$$
$$10x+4y-46 = 0$$
$$2(5x+2y-23) = 0$$
$$5x+2y-23 = 0/2$$
$$5x+2y-23 = 0$$

$\therefore$ Equation of the straight line is

$$5x+2y-23 = 0 \,//$$

* Given angle θ then slope $\boxed{m = \tan\theta}$

1. Find the equation of the line passing through the points $(2,3)$ having inclination 135°.

Sol Given point $(\underset{x_1, y_1}{2,3})$ and $\theta = 135^\circ$ $\Rightarrow$ slope $m = \tan\theta$

$= \tan 135^\circ$

$\boxed{m = -1}$

To find equation of straight line

We know that

equation of the straight line

$$y-y_1 = m(x-x_1)$$

$$y-3 = -1(x-2)$$

$y - 3 + x - 2 = 0$

$x + y - 5 = 0$ //

Find the value of 'x' if the slope of line joining points $(2,5), (x,3)$ is 2.

Sol Given points $\overset{x_1, y_1}{(2,5)}, \overset{x_2, y_2}{(x,3)}$

and slope $m = 2$

we know that

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{3-5}{x-2}$$

$$2 = \frac{-2}{x-2}$$

$$2(x-2) = -2$$

$$2x - 4 = -2$$

$$2x = -2 + 4$$

$$2x = 2$$

$$x = \frac{\cancel{2}}{\cancel{2}}$$

$$\boxed{x = 1}$$

H.W Find the equation of the straight line passing through the point $[1,2]$ and having inclination $\frac{\pi}{3}$ [A: $\sqrt{3}x - y - \sqrt{3} + 2$]

Given points $(1,2)$ and having inclination $\pi/3$

To find equation of straight line passing

$$\theta = \pi/3 = \frac{180}{3} = 60$$

$$\text{Slope } m = \tan\theta$$

$$= \tan(60)$$

$$m = \sqrt{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \sqrt{3}(x - 1)$$

$$y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\sqrt{3}x - y - \sqrt{3} + 2 = 0$$ //

16/12/19 * If parallel to x-axis then slope $\boxed{m=0}$

* If parallel to y-axis then slope $\boxed{m = 1/0}$

1. Find the equation of the line passing (3, -1) and parallel to y-axis.

Sol. Given point (3, -1)

and parallel to y-axis

Then slope $m = 1/0$

If point $(\overset{x_1}{3}, \overset{y_1}{-1})$ and slope $m = 1/0$

Then equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = 1/0(x-3) \;?$$

$$0 = x - 3$$

$$\therefore \boxed{x - 3 = 0}$$

H.W

2. Find the equation of the line passing (1, 2) and parallel to x-axis.
(y-2 = 0)

Sol Given point (1,2)

and parallel to x-axis

Then slope $m = 0$

If point $(\overset{x_1}{1}, \overset{y_1}{2})$ and slope $m = 0$

Then equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 0(x - 1)$$

$$\boxed{y - 2 = 0}$$

* Given slope 'm', y-intercept 'c' then the equation of the line is $\boxed{y = mx + c}$

1. If slope 4/7, and y-intercept 4 then find equation of the line.

Sol Given slope $m = 4/7$

and y-intercept $c = 4$

Then equation of the line is $y = mx + c$

$$y = 4/7\, x + 4$$

$$y = \frac{4x+28}{7}$$

$$7y = 4x+28$$

$$4x+28-7y=0$$

$$\Rightarrow 4x-7y+28=0$$

* If the line $ax+by+c=0$ then slope $\boxed{m = -a/b}$

1. If the line $2x+3y+7=0$ then find slope

Sol Given line $2x+3y+7=0$

$a=2,\ b=3$

To find slope

We know that

slope $m = -a/b$

$= -2/3$

* If parallel to the line $ax+by+c=0$ then slope

$\boxed{m = -a/b}$

* If perpendicular to the line $ax+by+c=0$ then slope

$\boxed{m = b/a}$

1. Find the equation of straight line passing through point (5,4) and parallel to line $2x+3y+7=0$

Sol Given point $(5,4)$ and parallel to line $2x+3y+7=0$

$a=2,\ b=3$

Then slope $m = -a/b$

$\boxed{\text{slope } m = -2/3}$

If point $(\overset{x_1}{5}, \overset{y_1}{4})$ and slope $m = -2/3$ then equation of the line is

$$y-y_1 = m(x-x_1)$$

$$y-4 = -2/3(x-5)$$

$$3(y-4) = -2(x-5)$$

$$3y-12 = -2x+10$$

$$3y-12+2x-10=0$$

$2x+3y-22=0$

H.W

2. Find the equation of straight line passing through point (4,-3) and perpendicular to line $5x-3y+1=0$ [Ans. $3x+5y+3=0$]

Sol Given point (4,-3) and ~~parallel~~ perpendicular to line $5x-3y+1=0$

$$a=5,\ b=-3$$

Then slope $m = b/a$

$$m = -3/5$$

If point $(\overset{x_1}{4}, \overset{y_1}{-3})$ and slope $m = -3/5$ then equation of the line is

$$y-y_1 = m(x-x_1)$$

$$y-(-3) = -3/5(x-4)$$

$$y+3 = -3/5(x-4)$$

$$5(y+3) = -3(x-4)$$

$$5y+15 = -3x+12$$

$$5y+15+3x-12 = 0$$

$$3x+5y+3=0$$

3. Find the equation of the line inclination 45° and y-intercep '-2'. $(x-y-2=0)$

Sol Given intercept $C = -2$

and $\theta = 45^\circ$

Slope $m = \tan\theta$

$$= \tan 45^\circ = 1$$

To find equation of line we know that

$$y = mx+c$$

$$y = 1(x)+(-2)$$

$$y = x-2$$

$$x-2-y=0$$

$$x-y-2=0$$

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* Given x-intercept 'a', y-intercept 'b' the equation [of] the line is $\boxed{\frac{x}{a}+\frac{y}{b}=1}$

1. If x-intercept 3, y-intercept 5 then find the equatio[n of] the line.

Sol Given x-intercept $a=3$

y-intercept $b=5$

Then equation of the line is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{3}+\frac{y}{5}=1$$

$$\Rightarrow \frac{5x+3y}{15}=1$$

$$\Rightarrow 5x+3y=15$$

$$\Rightarrow 5x+3y-15=0$$ //

2. Find the equation of the line having intercept a, b on th[e] axis such that $a+b=5$, $ab=6$

Sol Given x-intercept 'a'

y-intercept 'b'

Then equation of the line is

$$\frac{x}{a}+\frac{y}{b}=1 \longrightarrow (1)$$

Given $a+b=5 \longrightarrow (2)$

$ab=6 \longrightarrow (3)$

$a=6/b$

Substitute 'a' value in eq (2), we get

$$\frac{6}{b}+b=5$$

$$\frac{6+b^2}{b}=5$$

$$6+b^2=5b$$

$$b^2-5b+6=0$$

$$b^2-3b-2b+6=0$$

$$b(b-3)-2(b-3)=0$$

$1\times 6=6$

$-3 \quad -2$

$$(b-3)(b-2)=0$$

$b-3=0 \Rightarrow b=3$ | $b-2=0 \Rightarrow b=2$

Step-1. Substitute $b=3$ in eq ③, we get

$$a = \cancel{6}^{2}/\cancel{3}$$

$$\boxed{a=2}$$

$$\therefore a=2,\ b=3$$

Then equation of the line is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{2}+\frac{y}{3}=1$$

$$\Rightarrow \frac{3x+2y}{6}=1$$

$$\Rightarrow 3x+2y=6$$

$$\Rightarrow 3x+2y-6=0$$

Step-2

Substitute $b=2$ in eq ③, we get

$$a=6/b \quad a=\cancel{6}^{3}/\cancel{2}$$

$$\boxed{a=3}$$

$$\therefore a=3,\ b=2$$

Then equation of the line is

$$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{3}+\frac{y}{2}=1$$

$$\Rightarrow \frac{2x+3y}{6}=1$$

$$\Rightarrow 2x+3y=6$$

$$\Rightarrow 2x+3y-6=0 \ //$$

8. * If perpendicular to the line $ax+by+c=0$ then equation of the line is $bx-ay+k=0$.

1. Find equation of the line whose x-intercept -2 and which is perpendicular to $2x+3y+5=0$.

Sol Given x-intercept $a=-2$

Perpendicular line $2x+3y+5=0$

Then equation of the line is

$3x-2y+k=0 \longrightarrow ①$ $\left[\because \frac{x}{a}+\frac{y}{b}\right]=1$

$3x-2y=-k$

Divide '$-k$' on both sides, we get

$$\frac{3x}{-k}-\frac{2y}{-k}=\frac{-k}{-k}$$

$$\frac{x}{(-k/3)}-\frac{y}{(-k/2)}=1$$

x intercept $a=-k/3$

$\Rightarrow \cancel{-}2=\cancel{-}k/3$

$\therefore k=6$

Substitute 'k' value in eq ①, we get

$3x-2y+6=0$ //.

Find the value of 'k', if three lines $x-y+1=0$, $x+y+1=$ $kx-5y-7=0$ are concurrent.

Sol Given

$x-y+1=0 \longrightarrow ①$

$x+y+1=0 \longrightarrow ②$

$kx-5y-7=0 \longrightarrow ③$ are concurrent.

To find 'k'

Solve eq ① & ②

$$\begin{array}{cccc} & x & y & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{array}$$

$|ad-bc|$

$$\frac{x}{-1-1}=\frac{y}{1-1}=\frac{1}{1+1}$$

$$\frac{x}{-2}=\frac{y}{0}=\frac{1}{2}$$

$\frac{x}{-2}=\frac{1}{2}$ | $y/0=1/2$

$x=-\frac{\cancel{2}}{\cancel{2}}$ | $y=0/2$

$\boxed{x=-1}$ | $y=0$

Substitute $x=-1$, $y=0$ in eq ③, we get

$k(-1)-5(0)-7=0$

$-k-7=0$

$\boxed{k=-7}$

Find the equation of the line passing through the point of intersection of lines $x-2y-3=0$, $x+3y-6=0$ and parallel to $3x+4y=7$.

Sol Given lines $x-2y-3=0 \longrightarrow ①$

$x+3y-6=0 \longrightarrow ②$

and parallel line $3x+4y=7$

Then slope $m=-a/b$

slope $m=-3/4$

Solve eq ① & ②, we have

$$\begin{array}{cccc} & x & y & 1 \\ -2 & -3 & 1 & -2 \\ 3 & -6 & 1 & 3 \end{array}$$

$$\frac{x}{12+9}=\frac{y}{-3+6}=\frac{1}{3+2}$$

$$\frac{x}{21}=\frac{y}{3}=\frac{1}{5}$$

$$\frac{x}{21}=\frac{1}{5} \qquad\qquad \frac{y}{3}=\frac{1}{5}$$

$$x=\frac{21}{5} \qquad\qquad y=3/5$$

If point $(21/5, 3/5)$ and slope $m=-3/4$ then equation of the line is

$$y-y_1=m(x-x_1)$$

$$y-3/5=\frac{-3}{4}(x-21/5)$$

$$\frac{5y-3}{\cancel{5}}=\frac{-3}{4}\left(\frac{5x-21}{\cancel{5}}\right)$$

$$4(5y-3)=-3(5x-21) \longrightarrow 20y-12=-15x+63$$

$$20y-12+15x-63=0$$

$$15x+20y-75=0 \qquad \therefore\ 3x+4y-15=0$$

$$5(3x+4y-15)=0$$

Find the equation of the line passing through the point of intersection of lines $x+3y-1=0$, $x-2y+4=0$ and perpendic[ular to]
$2x+3y=0$ Ans. $[3x-2y+8=0]$

Sol: Given lines

$$x+3y-1=0 \quad \text{——} ①$$
$$x-2y+4=0 \quad \text{——} ②$$

and perpendicular line $2x+3y=0$

Then slope $m = b/a$
$= 3/2$

Solve eq ① & ②, we have

$$\frac{x}{\begin{matrix}3 & -1\\ -2 & 4\end{matrix}} = \frac{y}{\begin{matrix}1 & 1\end{matrix}} = \frac{1}{\begin{matrix}3\\ -2\end{matrix}}$$

$$\frac{x}{12-2} = \frac{y}{-1-4} = \frac{1}{-2-3}$$

$$\frac{x}{10} = \frac{y}{-5} = \frac{1}{-5}$$

$\frac{x}{10} = \frac{1}{-5}$ | $\frac{y}{-5} = \frac{1}{-5}$

$x = \frac{\cancel{10}^{2}}{\cancel{-5}}$ | $y = \frac{\cancel{-5}^{1}}{\cancel{-5}}$

$x = -2$ | $y = 1$

If point $(-2,1)$ and slope $m = \frac{3}{2}$ then equation of the l[ine]

$$y-y_1 = m(x-x_1)$$
$$y-1 = \frac{3}{2}(x-(-2))$$
$$2(y-1) = 3(x+2)$$
$$2y-2 = 3x+6$$
$$3x+6-2y+2=0$$
$$3x-2y+8=0 \;//$$

18/12/2019 The lines $2x-ay+1=0$, $3x-by+1=0$, $4x-cy+1=0$ are concurrent then show that a, b, c are in Arithematic progres[sion] (A.P)

Sol Given lines

$$2x-ay+1=0 \longrightarrow ①$$

$3x - by + 1 = 0 \longrightarrow ②$

$4x - cy + 1 = 0 \longrightarrow ③$ are concurrent

To show that a, b, c are A.P

It is enough to prove that $2b = a + c$

The determinant of co-efficient matrix of lines is zero

$$\begin{vmatrix} \overset{+}{2} & \overset{-}{-a} & \overset{+}{1} \\ 3 & -b & 1 \\ 4 & -c & 1 \end{vmatrix} = 0$$

$$2(-b+c) + a(3-4) + 1(-3c+4b) = 0$$

$$-2b + 2c - a - 3c + 4b = 0$$

$$2b - c - a = 0$$

$$2b = a + c$$

a, b, c are in Arithematic progression.

Angle between two lines

* Given two lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ then

$$\cos\theta = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}$$

1. If acute angle between two lines $4x - y + 7 = 0$, $kx - 5y - 9 = 0$ is 45° Then find the value of 'k'.

Sol Given lines $4x - y + 7 = 0$

$a_1 = 4, b_1 = -1, c_1 = 7$

$kx - 5y - 9 = 0$

$a_2 = k, b_2 = -5, c_2 = -9$

and $\theta = 45^\circ$

We know that

$$\cos\theta = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}$$

$$\cos 45 = \frac{|4k + (-1)(-5)|}{\sqrt{16+1}\sqrt{k^2+25}}$$

$$\frac{1}{\sqrt{2}} = \frac{|4k+5|}{\sqrt{17}\sqrt{k^2+25}}$$

Squaring on both sides, we get

$$\frac{1}{2} = \frac{(4k+5)^2}{17(k^2+25)} \qquad \left[(a+b)^2 = a^2+2ab+b^2\right]$$

$$17(k^2+25) = 2\left[16k^2+2(4k)(5)+25\right]$$

$$17k^2+425 = 32k^2+80k+50$$

$$32k^2+80k+50-17k^2-425=0$$

$$15k^2+80k-375=0$$

$$5(3k^2+16k-75)=0$$

$$3k^2+16k-75=0$$

$3 \times -75 = -225$: 25, -9

$$3k^2+25k-9k-75=0$$

$$3k^2-9k+25k-75=0$$

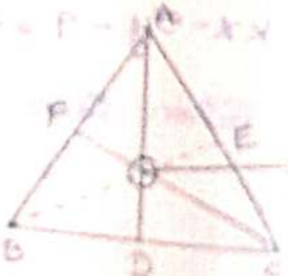
$$3k(k-3)+25(k-3)=0$$

$$(k-3)(3k+25)=0$$

$k-3=0$, $\boxed{k=3}$ | $3k+25=0$, $3k=-25$, $\boxed{k=-25/3}$

2. Find the acute angle between two lines $3x+5y=7$, $2x-y+4=0$ $\left[\cos\theta = \frac{1}{\sqrt{170}},\ \theta = \cos^{-1}\left(\frac{1}{\sqrt{170}}\right)\right]$

Orthocentre:



* If 'C' point and slope $CF = \frac{-1}{\text{Slope } AB}$ then equation of straight line
* If 'A' point and slope $AD = \frac{-1}{\text{Slope } BC}$ then equation of straight line

3. Find the orthocentre of triangle by the points $(-5,-7)$, $(13,2)$, $(-5,6)$

Sol: Given point $A=(-5,-7)$, $B=(13,2)$, $C=(-5,6)$

To find orthocentre

If $C=(-5,6)$ and slope $CF = \frac{-1}{\text{Slope } AB} \to (1)$

Consider slope $AB = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{2-(-7)}{13-(-5)}$

$= \cancel{9}/\cancel{18}\,2$

Slope $AB = \frac{1}{2}$

From eq ① becomes

slope $CF = -1/\tfrac{1}{2} = -2$

If $C = (-5,6)$ and slope $CF = -2$ then equation of lines

$y - y_1 = m(x - x_1)$

$y - 6 = -2x - 10$

$y - 6 + 2x + 10 = 0$

$2x + y + 4 = 0 \longrightarrow$ ②

If 'A' $= (-5,-7)$ and slope $AD = \dfrac{-1}{\text{Slope } BC} \longrightarrow$ ③

consider slope $BC = \dfrac{y_2-y_1}{x_2-x_1}$

$9x - 2y + 31 = 0 \rightarrow$

$x, y = (-3, 2)$

$= \dfrac{6-2}{-5-13}$

$= \dfrac{\cancel{4}\,2}{-\cancel{18}\,9}$

slope $BC = \dfrac{-2}{9}$

From eq ① becomes

slope $BC = \dfrac{-1}{-2/9} = \dfrac{\cancel{-}9}{\cancel{-}2} = \dfrac{9}{2}$

If 'A' $= (-5,-7)$ and slope $AD = \frac{9}{2}$ then equation of lines

$y - y_1 = m(x - x_1)$

$y - (-7) = \frac{9}{2}(x - (-5))$

$y + 7 = \frac{9}{2}(x + 5)$

$2(y + 7) = 9(x + 5)$

$2y + 14 = 9x + 45$

$9x + 45 - 2y - 14 = 0$

$9x - 2y + 31 = 0 \longrightarrow$ ④

~~~~ From eq ② & ④
~~~~

$2x+y+4=0$

$9x-2y+31=0$

$$\begin{array}{cccc} & x & y & 1 \\ 1 & 4 & 2 & 1 \\ -2 & 31 & 9 & -2 \end{array}$$

$$\frac{x}{31+8} = \frac{y}{36-62} = \frac{1}{-4-9}$$

$$\frac{x}{39} = \frac{y}{-26} = \frac{1}{-13}$$

$$\frac{x}{39} = \frac{1}{-13} \quad \Big| \quad \frac{y}{-26} = \frac{1}{-13}$$

$$x = \frac{\cancel{39}\,3}{\cancel{-13}} \quad \Big| \quad y = \frac{\cancel{-26}\,2}{\cancel{-13}}$$

$$x = -3, \quad y = 2$$

Find the acute angle between two lines $3x+5y=7$, $2x-y+4=0$

Sol Given lines

$$3x+5y=7$$
$$a_1=3, \; b_1=5, \; c_1=7$$
$$2x-y+4=0$$
$$a_2=2, \; b_2=-1, \; c_2=4$$

We know that

$$\cos\theta = \frac{|a_1a_2+b_1b_2|}{\sqrt{a_1^2+b_1^2}\,\sqrt{a_2^2+b_2^2}}$$

$$\cos\theta = \frac{3\times2+5\times-1}{\sqrt{(3)^2+(5)^2}\,\sqrt{(2)^2+(-1)^2}}$$

$$\cos\theta = \frac{6-5}{\sqrt{9+25}\,\sqrt{4+1}}$$

$$\cos\theta = \frac{1}{\sqrt{31}\,\sqrt{5}}$$

Squaring on both sides, we get

$$\cos^2\theta = \frac{1}{(\sqrt{31})^2(\sqrt{5})^2}$$

$$\cos^2\theta = \frac{1}{31\times5}$$

$$\cos^2\theta = \frac{1}{\ldots}$$

$$\cos\theta = \frac{1}{\sqrt{170}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{170}} \quad //$$

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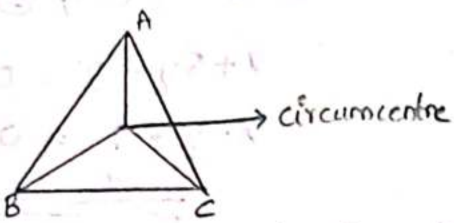
Circum Centre

$SA = SB = SC$



1. Find the circumcentre of the triangle formed by the points $(1,3), (0,-2), (-3,1)$

Sol Given points $A = (\overset{x_2}{1}, \overset{y_2}{3})$, $B = (\overset{x_1}{0}, \overset{y_2}{-2})$, $C = (\overset{x_2}{-3}, \overset{y_2}{1})$

let $S = (\overset{x_1}{x}, \overset{y_1}{y})$ be the circum centre point

We know that

$SA = SB = SC$ $\quad \left[\because \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right]$

consider

$SA = SB$

$$\sqrt{(1-x)^2+(3-y)^2} = \sqrt{(0-x)^2+(-2-y)^2}$$

Squaring on both sides

$$1-2(1)(x)+x^2+9-2(3)(y)+y^2 = x^2+(-2)^2-2(-2)y+y^2$$

$$10-2x+\cancel{x^2}-6y+\cancel{y^2} = \cancel{x^2}+4+4y+\cancel{y^2}$$

$$4+4y-10+2x+6y = 0$$

$$2x+10y-6=0$$

$$2(x+5y-3)=0$$

$$x+5y-3=0 \longrightarrow (1)$$

Again consider

$SB = SC$

$$\sqrt{(0-x)^2+(-2-y)^2} = \sqrt{(-3-x)^2+(1-y)^2}$$

Squaring on both sides

$$(-x)^2+(-2)^2-2(-2)(y)+(y)^2 = (-3)^2-2(-3)(x)+x^2+(1)^2-2(1)(y)+y^2$$

$$\cancel{x^2}+4+4y+\cancel{y^2} = 9+6x+\cancel{x^2}+1-2y+\cancel{y^2}$$

$$9+6x+1-2y-4-4y = 0$$

$$6x-6y+6=0$$

$$6(x-y+1)=0$$

$$x-y+1=0 \longrightarrow (2)$$

Solve eq (1) & (2)

$$x+5y-3=0$$

$$x-y+1=0$$

$$\begin{array}{cccc} x & y & 1 & \\ 5 & -3 & 1 & 5 \\ -1 & 1 & 1 & -1 \end{array}$$

$$\frac{x}{5-3}=\frac{y}{-3-1}=\frac{1}{-1-5}$$

$$\frac{x}{2}=\frac{y}{-4}=\frac{1}{-6}$$

$$\frac{x}{2}=\frac{1}{-6} \qquad \frac{y}{-4}=\frac{1}{-6}$$

$$x=\frac{\cancel{2}1}{-\cancel{6}3} \qquad y=\frac{\cancel{4}2}{\cancel{6}3}$$

$$x=-\frac{1}{3} \qquad y=\frac{2}{3}$$

$$\therefore x=-1/3,\ y=2/3$$

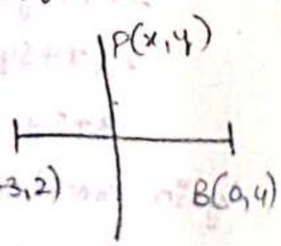
Locus

1. Find the equation of locus of point equidistance from the points $(-3,2)$, $(0,4)$

$6x+4y-3=0$

Sol Given points $A=(\overset{x_1,y_1}{-3,2})$, $B=(\overset{x_2,y_2}{0,4})$

let $P=(\overset{x,y}{x,y})$ be the locus point



$$PA=PB$$

$$\sqrt{(-3-x)^2+(2-y)^2}=\sqrt{(0-x)^2+(4-y)^2}$$

Squaring on both sides

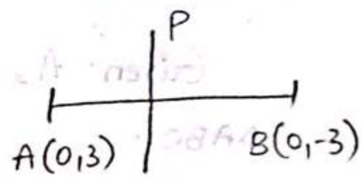
$$(-3)^2-2(-3)(x)+(x)^2+(2)^2-2(2)(y)+y^2=(-x)^2+(4)^2-(2)(4)(y)+y^2$$

$$9+6x+\cancel{x^2}+4-4y+\cancel{y^2}=\cancel{x^2}+16-8y+\cancel{y^2}$$

$$9+6x+4-4y-16+8y=0$$

$6x + 4y - 3 = 0$ //

2. Find the locus of 'p' if the distance of 'p' from $(0,3)$ is twice the distance of $(0,-3)$



Sol Given points $A = \overset{x_2\ y_2}{(0,3)}$, $B = \overset{x_2\ y_2}{(0,-3)}$

let 'P' $= \overset{x_1\ y_1}{(x, y)}$ be the locus of point

By data

$PA = 2\,PB$ $\quad x^2+y^2+10y+9=0$

$$\sqrt{(0-x)^2+(3-y)^2} = 2\left[\sqrt{(0-x)^2+(-3-y)^2}\right]$$

Squaring on both sides

$$(-x)^2+(3)^2-2(3)(y)+y^2 = 4\left[(-x)^2+(-3)^2-2(-3)(y)+y^2\right]$$

$$x^2+9-6y+y^2 = 4\left[x^2+9+6y+y^2\right]$$

$$x^2+9-6y+y^2 = 4x^2+36+24y+4y^2$$

$$4x^2+36+24y+4y^2-x^2-9+6y-y^2 = 0$$

$$3x^2+3y^2+30y+27 = 0$$

$$3(x^2+y^2+10y+9) = 0$$

$$x^2+y^2+10y+9 = 0 \quad //$$

3. Find the locus of point which is distance 5 unit from $(-3,4)$.

Sol Given point $A = \overset{x_2\ y_2}{(-3,4)}$, distance $= 5$ units

Let $P = \overset{x_1\ y_1}{(x, y)}$ be the locus of point $\quad x^2+6x-8y+y^2=0$

By data

$PA = 5$

$$\sqrt{(-3-x)^2+(4-y)^2} = 5$$

Squaring on both sides

$$(-3)^2-2(-3)(x)+x^2+(4)^2-2(4)(y)+y^2 = 25$$

$$9+6x+x^2+16-8y+y^2 = 25$$

$$x^2+6x-8y+y^2+25-25 = 0$$

$x^2+6x-8y+y^2=0$ ||

Incentre, Excentre

Given $A=(x_1,y_1)$, $B=(x_2,y_2)$, $C=(x_3,y_3)$ are vertices of ΔABC.

If $a=BC$, $b=AC$, $c=AB$, then

$$\text{Incentre } I=\left[\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right]$$

$$\text{Excentre opposite to } A = I_1 = \left[\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right]$$

$$\text{" " } B = I_2 = \left[\frac{ax_1-bx_2+cx_3}{a-b+c}, \frac{ay_1-by_2+cy_3}{a-b+c}\right]$$

$$\text{" " } C = I_3 = \left[\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c}\right]$$

1. Find the incentre, excentre of the triangle formed by the points $(3,2)$, $(7,2)$, $(7,5)$.

Sol Given points $A=\overset{x_1,y_1}{(3,2)}$, $B=\overset{x_2,y_2}{(7,2)}$, $C=\overset{x_3,y_3}{(7,5)}$

To find incentre, excentre.

We know that

$$\text{Incentre } I=\left[\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right]$$

$$\text{excentre opposite to } A = I_1 = \left[\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right]$$

$$B=I_2=\left[\frac{ax_1-bx_2+cx_3}{a-b+c}, \frac{ay_1-by_2+cy_3}{a-b+c}\right]$$

$$C=I_3=\left[\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c}\right]$$

$a = BC$

$a = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$a = \sqrt{(7-7)^2+(5-2)^2}$

$a = \sqrt{(0)^2+(3)^2}$

$a = \sqrt{9}$

$a = 3$

$a=3, b=5, c=4$

$I = (6,3)$

$I_1 = (9,4)$

$I_2 = (1,8)$

$I_3 = (4,-1)$

$b = AC$

$b = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$b = \sqrt{(7-3)^2+(5-2)^2}$

$b = \sqrt{(4)^2+(3)^2}$

$b = \sqrt{16+9}$

$b = \sqrt{25}$

$b = 5$

$c = AB$

$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-3)^2+(2-2)^2}$

$= \sqrt{(4)^2+(0)^2}$

$= \sqrt{16}$

$= 4$

Incentre $I = \left[\frac{a_1x_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right]$

$= \left[\frac{3\times3+5\times7+4\times7}{3+5+4}, \frac{3\times2+5\times2+4\times5}{3+5+4}\right]$

$= \left[\frac{9+35+28}{12}, \frac{6+10+20}{12}\right]$

$$= \left[\frac{72}{12}, \frac{36}{12}\right]$$

$$I = [6, 3]$$

Excentre $I_1 = \left[\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right]$

$$= \left[\frac{-3\times3+5\times7+4\times7}{-3+5+4}, \frac{-3\times2+5\times2+4\times5}{-3+5+4}\right]$$

$$= \left[\frac{-9+35+28}{-3+9}, \frac{-6+10+20}{-3+9}\right]$$

$$= \left[\frac{-9+63}{6}, \frac{-6+30}{6}\right]$$

$$= \left[\frac{54}{6}, \frac{24}{6}\right]$$

$$I_1 = [9, 4]$$

$$I_2 = \left[\frac{ax_1-bx_2+cx_3}{a-b+c}, \frac{ay_1-by_2+cy_3}{a-b+c}\right]$$

$$= \left[\frac{3\times3-5\times7+4\times7}{3-5+4}, \frac{3\times2-5\times2+4\times5}{3-5+4}\right]$$

$$= \left[\frac{9-35+28}{7-5}, \frac{6-10+20}{7-5}\right]$$

$$= \left[\frac{37-35}{2}, \frac{26-10}{2}\right]$$

$$= \left[\frac{\cancel{2}1}{\cancel{2}}, \frac{\cancel{16}8}{\cancel{2}}\right]$$

$$I_2 = [1, 8]$$

$$I_3 = \left[\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c}\right]$$

$$= \left[\frac{3\times3+5\times7-4\times7}{3+5-4}, \frac{3\times2+5\times2-4\times5}{3+5-4}\right]$$

$$= \left[\frac{9+35-28}{8-4}, \frac{6+10-20}{8-4}\right]$$

$$= \left[\frac{44-28}{4}, \frac{16-20}{4}\right]$$

$$= \left[\frac{\cancel{16}\,4}{\cancel{4}}, \frac{-\cancel{4}\,1}{\cancel{4}}\right]$$

$$I_3 = [4, -1]$$

26/10/2023
Thursday

Formula :-

1. $\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$

Ex: 1. $\int x^5 \cdot dx = \frac{x^{5+1}}{5+1} = \frac{x^6}{6}$

2. $\int \frac{1}{x^4} \cdot dx = \int x^{-4} \cdot dx = \frac{x^{-4+1}}{-4+1}$

2. $\int x \cdot dx = \frac{x^2}{2}$

2. $\int dx = x$

3. $\int 1/x \cdot dx = \log x$

4. $\int \log x \, dx = x \log x - x$

5. $\int \sin x \, dx = -\cos x$

6. $\int \sin ax \cdot dx = -\frac{\cos ax}{a}$

Ex: $\int \sin 2x \cdot dx = -\frac{\cos 2x}{2}$

7. $\int \cos x \cdot dx = \sin x$

8. $\int \cos ax \cdot dx = \frac{\sin ax}{a}$

Ex: $\int \cos 2x \cdot dx = \frac{\sin 2x}{2}$

9. $\int \tan x \, dx = \log \sec x$

10. $\int \cot x \, dx = \log \sin x$

11. $\int \sec^2 x \, dx = \tan x$

12. $\int \sec x \tan x \cdot dx = \sec x$

13. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x$

14. $\int \frac{1}{1+x^2} \cdot dx = \tan^{-1} x$

15. $\int \frac{1}{x^2+a^2} \, dx = \int \frac{1}{a^2+x^2} \cdot dx = \frac{1}{a} \tan^{-1}(x/a)$

Ex:- 1. $\int \frac{1}{x^2+4} \cdot dx = \int \frac{1}{x^2+(2)^2} \cdot dx = \frac{1}{2} \tan^{-1}(x/2)$

2. $\int \frac{1}{x^2+3} \cdot dx = \int \frac{1}{x^2+(\sqrt{3})^2} \cdot dx = \frac{1}{\sqrt{3}} \tan^{-1}(x/\sqrt{3})$

16. $\int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1}(x/a)$

17. $\int e^x \cdot dx = e^x$

18. $\int e^{ax} \cdot dx = \frac{e^{ax}}{a}$

Ex:- 1. $\int e^{3x} \cdot dx = \frac{e^{3x}}{3}$

2. $\int e^{-5x} \cdot dx = \frac{e^{-5x}}{-5}$

19. $\int e^{-x} dx = -e^{-x}$

20. $\int uv dx = uv_1 - u'v_2 + u''v_3 - \cdots\cdots$

where u' - differentiation at one time
v_1 - Integration of one time.

Ex:- $\int \underset{u}{x} \underset{v}{e^x} \cdot dx$

$= \underset{\text{diff}}{(x)} \underset{\text{Integration}}{(e^x)} - \underset{\text{diff}}{(1)} \underset{\text{Integration}}{(e^x)} + 0$

$= x \cdot e^x - e^x$ //

21. $\int$ constant $\cdot dx =$ constant$\cdot x$

Ex:- $\int 5 dx = 5x$

Trigonometric Formula

1. $\sin 2x = 2 \sin x \cos x$

2. $\cos^2 x = \frac{1+\cos 2x}{2}$, Ex: $\cos^2 2x = \frac{1+\cos 4x}{2}$

3. $\sin^2 x = \frac{1-\cos 2x}{2}$

4. $\sin^3 x = \frac{1}{4}[3 \sin x - \sin 3x]$

5. $\cos^3 x = \frac{1}{4}\left[3\cos x + \cos 3x\right]$

1. Evaluate $\int \frac{x^2+2x+2}{x^4} \cdot dx$

Sol Given

$$\int \frac{x^2+2x+2}{x^4} \cdot dx$$

$$= \int\left(\frac{\cancel{x^2}}{x^{\cancel{4}2}} + \frac{2\cancel{x}}{x^{\cancel{4}3}} + \frac{2}{x^4}\right) \cdot dx$$

$$= \int (x^{-2} + 2x^{-3} + 2x^{-4})\, dx \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{x^{-2+1}}{-2+1} + 2\frac{x^{-3+1}}{-3+1} + 2\frac{x^{-4+1}}{-4+1}$$

$$= \frac{x^{-1}}{-1} + \cancel{2}\frac{x^{-2}}{-\cancel{2}} + 2\frac{x^{-3}}{-3}$$

$$= -x^{-1} - x^{-2} - \frac{2}{3}x^{-3} + c$$

2. Evaluate $\int (x + 1/x)^3\, dx$.

Sol Given

$$\int (x+1/x)^3\, dx \qquad \left[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\right]$$

$$= \int \left[x^3 + 3x^{\cancel{2}}(1/\cancel{x}) + 3\cancel{x}(1/x)^{\cancel{2}} + (1/x)^3\right] dx$$

$$= \int \left[x^3 + 3x + 3(1/x) + 1/x^3\right] \cdot dx \qquad \left[\int 1/x \cdot dx = \log x\right]$$

$$= \int \left[x^3 + 3x + 3(1/x) + x^{-3}\right] \cdot dx \qquad \left[\int x^n\, dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{x^4}{4} + 3\frac{x^2}{2} + 3\log x + \frac{x^{-3+1}}{-3+1}$$

$$= \frac{x^4}{4} + \frac{3}{2}x^2 + 3\log x - \frac{x^{-2}}{2} + c$$

3. Evaluate $\int \sqrt{1+\sin 2x}\,dx$

Sol Given

$$\int \sqrt{1+\sin 2x}\cdot dx \qquad [\sin^2 x+\cos^2 x=1]$$

$$=\int \sqrt{1+2\sin x\cdot\cos x}\cdot dx$$

$$=\int \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\cdot dx \qquad \left[a^2+b^2+2ab=(a+b)^2,\ a=\sin x,\ b=\cos x\right.$$

$$=\int \sqrt{(\sin x+\cos x)^{\cancel{2}}}\cdot dx \qquad \int \sin x\cdot dx=-\cos x$$

$$=\int (\sin x+\cos x)\cdot dx \qquad \int \cos x\cdot dx=\sin x$$

$$=-\cos x+\sin x+c$$

4. Evaluate $\int \sqrt{1+\cos 2x}\,dx$.

Sol Given

$$\int \sqrt{1+\cos 2x}\cdot dx.$$

$$\left[\cos^2 x=\frac{1+\cos 2x}{2}\right.$$

$$\left.2\cos^2 x=1+\cos 2x\right]$$

$$=\int \sqrt{2\cdot\cos^2 x}\cdot dx$$

$$=\int \sqrt{2}\,\sqrt{\cos^{\cancel{2}} x}\cdot dx$$

$$=\sqrt{2}\int \cos x\cdot dx \qquad \left[\int \cos x\cdot dx=\sin x\right]$$

$$=\sqrt{2}\sin x+c$$

H.W

5. Evaluate $\int \sqrt{1-\cos 2x}\cdot dx$.

Sol Given

$$\int \sqrt{1-\cos 2x}\cdot dx \qquad \sin^2 x=\frac{1-\cos 2x}{2}$$

$$2\sin^2 x=1-\cos 2x$$

$$=\int \sqrt{2\cdot\sin^2 x}\cdot dx$$

$$=\int \sqrt{2}\,\sqrt{\sin^{\cancel{2}} x}\cdot dx$$

$= \sqrt{2} \int \sin \cdot dx$

$= \sqrt{2} - \cos x$

$= \sqrt{2} - \cos x + C$ ||

6. Evaluate $\int \cos^4 x \cdot dx$

Sol Given

$\int \cos^4 x \cdot dx$

$= \int (\cos^2 x)^2 dx$

$= \int \left[\frac{1+\cos 2x}{2}\right]^2 dx$

$= \int \frac{(1+\cos 2x)^2}{4} \cdot dx$

$= \frac{1}{4}\int \left[(1)^2 + 2(1)(\cos 2x) + \cos^2 2x\right] dx$

$= \frac{1}{4}\int \left[1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right]$

$= \frac{1}{4}\left[1x + \cancel{2}\frac{\sin \cdot 2x}{\cancel{2}} + 1/2\left(1x + \frac{\sin 4x}{4}\right)\right]$

$= \frac{1}{4}\left[x + \sin 2x + 1/2\left(x + \frac{\sin 4x}{4}\right)\right] + C$ ||

M.

7. Evaluate i. $\int \sin^2 x \, dx$ (ii) $\int \cos^2 x \, dx$

11.10

iii. $\int \sin^3 x \, dx$ (iv) $\int \cos^3 x \, dx$

8. Integration w.r to x: $2x\sqrt{x}$ (or) Find $\int 2x\sqrt{x} \, dx$.

Sol Given $\int 2x^1\sqrt{x} \cdot dx$

$= 2\int x^1 \cdot x^{1/2} dx$ $\quad [1 + 1/2 = 3/2]$

$= 2\int x^{3/2} \cdot dx$ $\quad \int x^n dx = \frac{x^{n+1}}{n+1}$

$= 2 \dfrac{x^{3/2+1}}{3/2+1}$

$= 2 \dfrac{x^{5/2}}{5/2}$

$= 2 \cdot x^{5/2} \, x^{2/5}$

$= 4/5 \cdot x^{5/2} + c$ //

7.8 Evaluate

i. $\int \sin^2 x \cdot dx$

Sol Given

$\int \sin^2 x \cdot dx$

$= \int \dfrac{1 - \cos 2x}{2} \cdot dx$ $\quad \left[\int \cos 2x \cdot dx = \dfrac{\sin 2x}{2}\right]$

$= \int \dfrac{1}{2} - \dfrac{\cos 2x}{2} \cdot dx$

$= \dfrac{1}{2} x - \dfrac{1}{2} \dfrac{\sin 2x}{2} + c$ //

ii. $\int \cos^2 x \cdot dx$

Sol Given $\int \cos^2 x \cdot dx$ $\quad \int \text{constant } dx = \text{constant } x$

$= \int \dfrac{1 + \cos 2x}{2} \cdot dx$

$= \int \dfrac{1}{2} + \dfrac{\cos 2x}{2} \cdot dx$

$= \int \dfrac{1}{2} + \dfrac{1}{2} \cdot \cos 2x \cdot dx$

$= \dfrac{1}{2} x + 1/2 \dfrac{\sin 2x}{2} + c$ //

iii. $\int \sin^3 x \, dx$

Sol Given $\int \sin^3 x \, dx$

$\int \dfrac{1}{4} [3 \sin x - \sin 3x] \, dx$

$$= \frac{1}{4}\left[3(-\cos x) + \frac{\cos 3x}{3}\right] + C \qquad \left[\int \sin x \cdot dx = -\cos x\right]$$
$$\left[\int -\sin 3x = -\frac{\cos 3x}{3}\right]$$

$$= \frac{1}{4}\left[3(-\cos x) + \frac{\cos 3x}{3}\right] + C \;//$$

$$= \frac{1}{4}\left[3 - \cos x + \frac{\cos 3x}{3}\right] + C \;//$$

iv. $\int \cos^3 x \cdot dx$

Sol Given

$$\int \cos^3 x \cdot dx$$

$$= \int \tfrac{1}{4}\left[3\cos x + \cos 3x\right] dx$$

$$= \frac{1}{4}\int \left[3\cos x + \cos 3x\right] dx$$

$$= \frac{1}{4}\left[3 \cdot \sin x + \frac{\sin 3x}{3}\right] + C \;//$$

9. Evaluate $\int (3\sec^2 x + 2\sin^2 x + e^{5x})\, dx$.

Sol Given

$$\int (3\sec^2 x + 2\sin^2 x + e^{5x})\, dx$$

$$= \int \left(3\sec^2 x + \cancel{2}\left(\frac{1-\cos 2x}{\cancel{2}}\right) + e^{5x}\right) \cdot dx$$

$$= 3\tan x + 1x - \frac{\sin 2x}{2} + e^{5x}/5 + C \;//$$

31/12/2019

Integration by substitution Method.

1. Evaluate $\int \sec^2 (2x+5)\, dx$

Sol Given

$$\int \sec^2 (2x+5)\, dx \longrightarrow (1)$$

Put $\boxed{t = 2x+5}$ $\qquad \left[\int \sec^2 x\, dx = \tan x\right]$

diff w.r.to x, we get

$$\frac{dt}{dx} = 2(1) + 0$$

$$\frac{dt}{dx} = 2$$

$$dt = 2dx$$

$$\boxed{\tfrac{1}{2}\,dt = dx}$$

From eq ① becomes

$$= \int \sec^2 t \; \tfrac{1}{2}\, dt$$

$$= \frac{1}{2}\int \sec^2 t \, dt$$

$$= \frac{1}{2} \tan t$$

Since $t = 2x+5$

$$= \frac{1}{2}\tan(2x+5) + c \;//$$

2. Evaluate $\int \frac{1}{5x+7} \cdot dx$ $\left[\frac{1}{5}\log(5x+7)+c\right]$

Sol Given $\int \frac{1}{5x+7} \cdot dx \longrightarrow ①$

Put $\boxed{t = 5x+7}$

diff. w.r.t x, we get

$$\frac{dt}{dx} = 5(1)+0$$

$$dt = 5\,dx$$

$$\boxed{\frac{1}{5}\,dt = dx}$$

From eq ① becomes

$$= \int \frac{1}{5x+7} \cdot \frac{1}{5}\,dt$$

$$= \frac{1}{5}\int \frac{1}{5x+7}\,dt \qquad \left[\int \frac{1}{x}\,dx = \log x\right]$$

$$= \frac{1}{5} \cdot \log t$$

Since $t = 5x+7$

$$= \frac{1}{5}\log(5x+7) + c \;//$$

3. Evaluate $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \cdot dx$

Sol Given $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \cdot dx \longrightarrow ①$

Put $\boxed{t = \sin^{-1}x}$

$$\int \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1}x$$

Diff w.r. to x, we get

$$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{dt = \frac{1}{\sqrt{1-x^2}} \cdot dx}$$

From eq ① becomes

$$= \int e^t dt$$

$$= e^t$$

Since $t = \sin^{-1}x$

$$= e^{\sin^{-1}x} + c$$

4. Evaluate $\int \frac{(\sin^{-1}x)^4}{\sqrt{1-x^2}} dx$

5. Evaluate $\int \frac{\cos(\log x)}{x} \cdot dx$

Sol Given

$$\int \frac{\cos(\log x)}{x} \cdot dx \longrightarrow ①$$

Put $\boxed{t = \log x}$

Diff. w.r. to x, we get

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\boxed{dt = 1/x \, dx}$$

From eq ① becomes

$$= \int \cos t \, dt$$

$$= \sin t$$

Since $t = \log x$

$$= \sin(\log x) + C.$$

6. Evaluate $\int \frac{e^{\log x}}{x} \cdot dx$ $\quad [e^{\log x}]$

7. Evaluate $\int \frac{\cos \sqrt{x}}{\sqrt{x}} \cdot dx$

Sol Given

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} \cdot dx \longrightarrow ①$$

Put $t = \sqrt{x}$

Diff w.r. to x, we get

$$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$$

$$dt = \frac{1}{2\sqrt{x}} dx$$

$$\boxed{2\,dt = \frac{1}{\sqrt{x}} dx}$$

From eq ① becomes

$$= \int \cos t \cdot 2dt$$

$$= 2 \int \cos t \cdot dt$$

$$= 2 \sin t$$

since $t = \sqrt{x}$

$$= 2 \sin \sqrt{x} + c$$

6. Evaluate $\int \frac{e^{\log x}}{x} \cdot dx.$

Sol Given $\int \frac{e^{\log x}}{x} \cdot dx \longrightarrow ①$

Put $\boxed{t = \log x}$

Diff. w.r.t x, we get

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\boxed{dt = \frac{1}{x} \cdot dx}$$

From eq ① becomes

$$= \int e^t . dt$$

$= e^t$

Since $t = \log x$

$= e^{\log x} + c$ //

8. Evaluate $\int \frac{x^3}{\sqrt{1-x^8}} \cdot dx$

Sol Given $\int \frac{x^3}{\sqrt{1-x^8}} \, dx$

$$= \int \frac{x^3}{\sqrt{1-(x^4)^2}} \cdot dx \quad \text{—— ①}$$

Put $\boxed{t = x^4}$

Diff w.r to x, we get

$$\frac{dt}{dx} = 4x^3$$

$$dt = 4x^3 \cdot dx$$

$$\boxed{\frac{1}{4} dt = x^3 \cdot dx}$$

From eq ① becomes

$$= \int \frac{1}{\sqrt{1-t^2}} \, \frac{1}{4} \cdot dt$$

$$= \frac{1}{4} \int \frac{1}{\sqrt{1-t^2}} \, dt \qquad \left[\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x \right.$$

$$= \frac{1}{4} \sin^{-1} t$$

Since $t = x^4$

$$= \frac{1}{4} \sin^{-1} x^4 + c \text{ //}$$

9. Evaluate $\int \frac{(\sin^{-1} x)^4}{\sqrt{1-x^2}} \, dx$

Sol Given $\int \frac{(\sin^{-1} x)^4}{\sqrt{1-x^2}} \, dx \longrightarrow ①$

Put $\boxed{t = \sin^{-1} x}$

Differentiation w.r.t $\cdot x$

$$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{dt = \frac{1}{\sqrt{1-x^2}} \cdot dx}$$

From eq ① becomes

$$\left[\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}\right]$$

$$\int t^4 . dt$$

$$= t^5/5$$

Since $t = \sin^{-1}x$ ①

$$= \frac{(\sin^{-1}x)^5}{5} + c$$

$$\therefore \int \frac{(\sin^{-1}x)^4}{\sqrt{1-x^2}} \cdot dx = \frac{(\sin^{-1}x)^5}{5} + c.$$

Formulas:

* $\int \frac{1}{a^2+x^2} \cdot dx = \frac{1}{a}\tan^{-1}(x/a)$
* $\int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1}(x/a)$
* $\int \frac{1}{\sqrt{a^2+x^2}} \cdot dx = \sinh^{-1}(x/a)$
* $\int \frac{1}{\sqrt{x^2-a^2}} dx = \cosh^{-1}(x/a)$
* $\int \sqrt{a^2-x^2}\, dx = x/2\sqrt{a^2-x^2} + a^2/2 \sin^{-1}(x/a)$
* $\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + a^2/2 \sinh^{-1}(x/a)$
* $\int \sqrt{x^2-a^2}\, dx = x/2\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}(x/a)$
* $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left[\frac{a+x}{a-x}\right]$

* $\int \frac{1}{x^2-a^2} \cdot dx = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)$

2/01/2020

1. Evaluate $\int \frac{1}{\sqrt{9-4x^2}} \cdot dx$

Sol Given $\int \frac{1}{\sqrt{9-4x^2}} \cdot dx$

$\left[\because \int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1}[x/a]\right]$

$= \int \frac{1}{\sqrt{4(9/4-x^2)}} \cdot dx$

$= \int \frac{1}{2\sqrt{(3/2)^2-x^2}} \cdot dx$

$= \frac{1}{2} \int \frac{1}{\sqrt{(3/2)^2-x^2}} \cdot dx \quad [a = 3/2]$

$= 1/2 \sin^{-1}\left(\frac{x}{3/2}\right)$

$= 1/2 \sin^{-1}\left[\frac{2x}{3}\right] + c$ //

2. Evaluate $\int \sqrt{16-9x^2} \cdot dx$

Sol Given $\int \sqrt{16-9x^2} \cdot dx$

$\left[\int \sqrt{a^2-x^2} \cdot dx = x/2 \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}(x/a)\right]$

$= \int \sqrt{9\left(\frac{16}{9}-x^2\right)} \cdot dx$

$= 3\int \sqrt{(4/3)^2-x^2} \cdot dx \quad [\because a = 4/3]$

$= 3\left[x/2 \sqrt{(4/3)^2-x^2} + \frac{(4/3)^2}{2} \sin^{-1}\left(\frac{x}{4/3}\right)\right]$

$$= 3\left[\frac{x}{2}\sqrt{\frac{16}{9}-x^2} + \frac{16}{9}\,x\,\frac{1}{\cancel{x}}\sin^{-1}\left(\frac{3x}{4}\right)\right]$$

$$= 3\left[\frac{x}{2}\sqrt{\frac{16-9x^2}{9}} + \frac{8}{9}\sin^{-1}\left(\frac{3x}{4}\right)\right]$$

$$= \frac{\cancel{3}x}{2}\frac{\sqrt{16-9x^2}}{\cancel{3}} + \cancel{3}\left(\frac{8}{\cancel{9}_{3}}\right)\sin^{-1}\left(\frac{3x}{4}\right)$$

$$= \frac{x}{2}\sqrt{16-9x^2} + \frac{8}{3}\sin^{-1}\left(\frac{3x}{4}\right) + c$$

Evaluate (i). $\int \frac{dx}{\sqrt{3+x^2}}$ ii. $\int \frac{dx}{\sqrt{3x^2-7}}$

iii. $\int \frac{1}{16+x^2}\cdot dx$.

Integral of the form $\int \frac{dx}{ax^2+bx+c}$, $\int \sqrt{ax^2+bx+c}\cdot dx$

M.

1. Evaluate $\int \frac{1}{3x^2+4x+1}\cdot dx$

Sol Given $\int \frac{1}{3x^2+4x+1}\cdot dx \longrightarrow (1)$

Consider $a = x$

$3x^2+4x+1 = 3\left[x^2+4/3\,x+1/3\right]$ $\left[\because a^2+2ab+b^2 = (a+b)^2\right.$

$a = x,\ b = 2/3$

$= 3\left[x^2+2(x)(2/3)+(2/3)^2-(2/3)^2+1/3\right]$ $2,\ x\ 2/3$

$= 3\left[(x+2/3)^2-\frac{4}{9}+\frac{1}{3}\right]$

$3\,\underline{|9,3}$

$3,1$

$= 3\left[(x+2/3)^2-\frac{(4-3)}{9}\right]$ $3\times3=9$

$= 3\left[(x+2/3)^2-1/9\right]$

$3x^2+4x+1 = 3\left[(x+2/3)^2-(1/3)^2\right]$

From eq (1) becomes

$$\int \frac{1}{3\left[(x+2/3)^2-(1/3)^2\right]}\cdot dx$$

$$= \frac{1}{3}\int \frac{1}{(x+2/3)^2-(1/3)^2}\cdot dx$$

$$\because \left[\int \frac{1}{x^2-a^2}\cdot dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)\right]$$

$$x = x+2/3 \;,\; a = 1/3$$

$$= \frac{1}{3}\left[\frac{1}{2(1/3)}\log\left(\frac{x+2/3-1/3}{x+2/3+1/3}\right)\right]$$

$$= \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{2}\log\left(\frac{\frac{3x+2-1}{\cancel{3}}}{\frac{3x+2+1}{\cancel{3}}}\right)$$

$$= \frac{1}{2}\log\left(\frac{3x+1}{3x+3}\right)+c$$

2. Evaluate $\int\sqrt{x^2+x+1}\cdot dx \longrightarrow ①$

Sol Given

$$\int\sqrt{x^2+x+1}\cdot dx \rightarrow ①$$

Consider

$$x^2+x+1 = x^2+2x(1/2)+(1/2)^2-(1/2)^2+1 \qquad \left[\begin{array}{c}a^2+2ab+b^2=(a+b)^2\\ a=x,\ b=\frac{1}{2}\end{array}\right]$$

$$= (x+1/2)^2-\frac{1}{4}+1$$

$$= (x+1/2)^2-\left(\frac{1-4}{4}\right)$$

$$= (x+1/2)^2+\frac{3}{4}$$

$$x^2+x+1 = (x+1/2)^2+\left(\frac{\sqrt{3}}{2}\right)^2$$

$$\left[\frac{2x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{9\sqrt{3}}\right)+c\right]$$

$$\left[\int\sqrt{x^2+a^2}\cdot dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\sinh^{-1}(x/a).\right]$$

$x = x+1/2,\ a = \sqrt{3}/2$

From eq ① becomes

$$\int\sqrt{x^2+x+1}\cdot dx = \int\sqrt{(x+1/2)^2+(\sqrt{3}/2)^2}\cdot dx$$

i. $\int \frac{dx}{\sqrt{3+x^2}}$

Sol Given

$$\int \frac{dx}{\sqrt{3+x^2}}$$

$$\left[\because \frac{1}{\sqrt{a^2+x^2}} \cdot dx = \sinh^{-1}(x/a)\right]$$

$$\int \frac{dx}{\sqrt{(\sqrt{3})^2+x^2}}$$ where $a=\sqrt{3}$, $x=x$

$$= \sinh^{-1}(x/\sqrt{3}) + c$$

iii. $\int \frac{1}{16+x^2} \cdot dx$

Sol Given $\int \frac{1}{16+x^2} \cdot dx$

$$\because \left[\frac{1}{a^2+x^2} \cdot dx = \frac{1}{a} \tan^{-1}(x/a)\right]$$

$$\int \frac{1}{(4)^2+x^2} \cdot dx$$ where $a=4$, $x=x$

$$= \frac{1}{4} \tan^{-1}(x/4) + c$$

7/01/2020 Integral of the form $\int \frac{1}{a \pm b\sin x}$ $\int \frac{1}{a \pm b\cos x}$

$$\int \frac{1}{a\sin x \pm b\cos x \pm c}$$

Put $t = \tan x/2$

$$dx = \frac{2}{1+t^2}\cdot dt$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

Evaluate $\int \frac{1}{5+4\sin x}\cdot dx$

Sol Given $\int \frac{1}{5+4\sin x}\cdot dx \longrightarrow ①$

Now compare $\int \frac{1}{a+b\sin x}\cdot dx$

Put $t = \tan x/2$

$$dx = \frac{2}{1+t^2}\cdot dt$$

$$\sin x = \frac{2t}{1+t^2}$$

From eq ① becomes

$$\int \frac{1}{5+4\left(\frac{2t}{1+t^2}\right)}\cdot \frac{2}{1+t^2}\cdot dt$$

$$\int \frac{1}{\frac{5(1+t^2)+8t}{1+t^2}}\cdot \frac{2}{1+t^2}\cdot dt$$

$$= \int \frac{1+t^2}{5+5t^2+8t}\cdot \frac{2}{1+t^2}\cdot dt$$

$$= 2\int \frac{1}{5t^2+8t+5}\, dt \rightarrow ②$$

$$\frac{2}{3}\tan^{-1}\left(\frac{5\tan x/2 + 4}{3}\right) + c$$

Consider

$$5t^2+8t+5 = 5\left(t^2 + \frac{8}{5}t + 1\right)$$

$$= 5\left(t^2 + 2(t)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 - \left(\frac{4}{5}\right)^2 + 1\right)$$

$$= 5\left((t+4/5)^2 - \frac{16}{25} + 1\right)$$

$$= 5\left((t+4/5)^2 - \frac{16+25}{25}\right)$$

$$= 5\left[(t+4/5)^2 - \frac{(-9)}{25}\right]$$

$$= 5\left[(t+4/5)^2 - \frac{(-9)}{25}\right]$$

$$= 5\left[(t+4/5)^2 + \left(\frac{3}{5}\right)^2\right]$$

From eq ② becomes

$$= 2\int \frac{1}{5(t+4/5)^2+(3/5)^2}\cdot dt$$

$$\left[\because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}(x/a)\right]$$

Here, $x = (t+4/5)$, $a = 3/5$

$$= \frac{2}{5}\left[\frac{1}{3/5}\cdot\tan^{-1}\left(\frac{t+4/5}{3/5}\right)\right]$$

$$= \frac{2}{\cancel{5}}\left[\cancel{5}/3\cdot\tan^{-1}\cancel{5}/3\cdot\left(\frac{5t+4}{\cancel{5}}\right)\right)$$

$$= \frac{2}{3}\tan^{-1} 1/3\,(5t+4)$$

$$= \frac{2}{3}\tan^{-1}\frac{1}{3}\left[5\tan\frac{x}{2}+4\right]$$

$$= \frac{2}{3}\tan^{-1}\left[\frac{5\tan x/2+4}{3}\right] + C$$

Evaluate (i). $\int \frac{1}{3+4\cos x}\cdot dx$

(ii) $\int \frac{1}{1+\sin x+\cos x}\cdot dx$

Sol. Given

$$\int \frac{1}{1+\sin x+\cos x}\cdot dx \longrightarrow ①$$

Now compare

$$\int \frac{1}{a+b\sin x+\cos x}\cdot dx$$

Put $t = \tan x/2$

$$dx = \frac{2}{1+t^2}\cdot dt$$

$$\sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2}$$

From eq ① becomes

$$\int \frac{1}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\cdot dt$$

7/01

$$= \int \frac{1}{\frac{\cancel{(1+t^2)}+2t+\cancel{1-t^2}}{(1+t^2)}} \cdot \frac{2}{1+t^2} \cdot dt$$

$$= \int \frac{\cancel{1+t^2}}{2t+2} \cdot \frac{2}{\cancel{1+t^2}} \cdot dt$$

$$= \cancel{2} \int \frac{1}{\cancel{2}(t+1)} \cdot dt$$

$$= \int \frac{1}{1+t} \cdot dt \qquad \left[\because \int \frac{1}{x} \cdot dx = \log x \quad \frac{d}{dt}(1+t) = 0+1 = 1 \right]$$

$$= \frac{\log (1+t)}{1}$$

$\therefore$ Since $t = \tan x/2$

$$= \log(1+\tan x/2) + C$$

Evaluate $\int \frac{\cos x}{2+3\sin x} \cdot dx$.

Sol Given $\int \frac{\cos x}{2+3\sin x} \cdot dx \longrightarrow (1)$

Put $\boxed{t = 2+3\sin x}$

Differentiation w.r.to x.

$$\frac{dt}{dx} = 0 + 3\cos x$$

$$dt = 3\cos x \cdot dx$$

$$\boxed{\frac{1}{3} dt = \cos x \cdot dx}$$

From eq (1) becomes

$$\int \frac{1}{t} \cdot \frac{1}{3} dt$$

$$= \frac{1}{3} \log t$$

Since $t = 2+3\sin x$

$$= \frac{1}{3} \log(2+3\sin x) + C$$

Evaluate $\int \frac{12x+22}{6x^2+22x+1} \cdot dx$

Sol Given

$$\int \frac{12x+22}{6x^2+22x+1} \cdot dx \longrightarrow (1)$$

Put $\boxed{t = 6x^2+22x+1}$

Differentiation w.r.t. to x

$$\frac{dt}{dx} = 6(2x)+22(1)+0$$

$$\frac{dt}{dx} = 12x+22$$

$$dt = 12x+22 \cdot dx$$

From eq (1) becomes

$$\int \frac{1}{t} \cdot 12x+22 \cdot dx$$

$$\int \frac{1}{t} \cdot dt$$

$$= \log t$$

Since $t = 6x^2+22x+1$

$$= \log(6x^2+22x+1) + c$$ //

9/01/2020 Method of partial fractions

Evaluate $\int \frac{2x+3}{3x^2+14x-5} \cdot dx$

Sol

$$\frac{2x+3}{3x^2+14x-5} = \frac{2x+3}{3x^2+15x-1x-5}$$

$$= \frac{2x+3}{3x(x+5)-(x+5)}$$

$$= \frac{2x+3}{(x+5)(3x-1)}$$

$$\frac{2x+3}{(x+5)(3x-1)} = \frac{2x+3}{3x^2+14x-5} \quad \text{①}$$

Consider

$$\frac{2x+3}{(x+5)(3x-1)} = \frac{A}{(x+5)} + \frac{B}{(3x-1)} \quad \text{②}$$

$$\frac{2x+3}{\cancel{(x+5)(3x-1)}} = \frac{A(3x-1)+B(x+5)}{\cancel{(x+5)(3x-1)}}$$

$$2x+3 = A(3x-1) + B(x+5) \quad \text{③}$$

To find A, B values

Find A

Put $x = -5$, we get

$$2(-5)+3 = A[3(-5)-1]+0$$

$$-10+3 = A[-15-1]$$

$$\cancel{-}7 = \cancel{-}16A$$

$$\boxed{7/16 = A}$$

From equation ③. To find B

Put $x = 1/3$, we get

$$2(1/3)+3 = 0+B(1/3+5)$$

$$\frac{2+9}{3} = B\left[\frac{1+15}{3}\right]$$

$$\frac{11}{\cancel{3}} = \frac{16}{\cancel{3}}B$$

$$\boxed{\frac{11}{16} = B}$$

Substitute A, B values in eq ②, we get

$$\frac{2x+3}{(x+5)(3x-1)} = \frac{7/16}{x+5} + \frac{11/16}{(3x-1)}$$

$$= \frac{7}{16(x+5)} + \frac{11}{16(3x-1)}$$

$$\int\left[\frac{7}{16(x+5)}+\frac{11}{16(3x-1)}\right]\cdot dx$$

$$=\frac{7}{16}\int\frac{1}{x+5}\cdot dx+\frac{11}{16}\int\frac{1}{3x-1}\cdot dx$$

$$\left[\int\frac{1}{x}dx=\log x \quad \frac{d}{dx}(x+5)=1 \quad \frac{d}{dx}(3x-1)=3\right]$$

$$=\frac{7}{16}\frac{\log(x+5)}{1}+\frac{11}{16}\frac{\log(3x-1)}{3}$$

$$=\frac{7}{16}\log(x+5)+\frac{11}{48}\log(3x-1)+c$$

Evaluate $\int\frac{3}{(x-1)(x+2)^2}$ into partial fractions.

Given $\int\frac{3}{(x-1)(x+2)^2}$ — (1)

Sol

$$\frac{3}{(x-1)(x+2)^2}=\frac{A}{(x-1)}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2} \quad \text{—— (1)}$$

$$\frac{3}{\cancel{(x-1)(x+2)^2}}=\frac{A(x+2)^2+B(x-1)(x+2)+C(x-1)}{\cancel{(x-1)(x+2)^2}}$$

$$3=A(x+2)^2+B(x-1)(x+2)+C(x-1) \longrightarrow (2)$$

To find A, B, C values

Put $x=1$, we get

$$3=A(1+2)^2+B(1-1)(1+2)+C(1-1)$$

$$3=A((1)^2+2(1)(2)+(2)^2)+0+0$$

$$3=A(1+4+4)$$

$$3=A(9)$$

$$\cancel{3}/\cancel{9}_3=A$$

$$1/3=A$$

$$\therefore \boxed{A=1/3}$$

From eq (2) put $x=-2$, we get

$$3=A(-2+2)^2+B(-2-1)(-2+2)+C(2-1)$$

$$3=0+0+C(-3)$$

$$3=-3C$$

$$\cancel{3}/-\cancel{3}=C$$

$$-1=C$$

$$\therefore \boxed{C=-1}$$

From eq ② simplify to find B

$3 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

$3 = A(x^2 + 2(x)(2) + (2)^2) + B(x^2 + 2x - x - 2) + Cx - C$

$3 = Ax^2 + 4Ax + 4A + Bx^2 + Bx - 2B + Cx - C$

Now compare 'x^2' co-efficient, we get

$0 = A + B$

$0 = \frac{1}{3} + B$

$\frac{-1}{3} = B$

$\therefore \boxed{B = \frac{-1}{3}}$

Substitute A, B, C values in eq ①, we get

$$\frac{3}{(x-1)(x+2)^2} = \frac{1/3}{(x-1)} + \frac{1/3}{(x+2)} + \frac{-1}{(x+2)^2}$$

$$= \frac{1}{3(x-1)} + \frac{(-1)}{3(x+2)} + \frac{(-1)}{(x+2)^2}$$

From eq ① becomes

$\left[\int \frac{1}{x^2} \cdot dx = \int x^{-2} \cdot dx = \frac{x^{-1}}{-1} = -1/x\right]$

$$\int \left[\frac{1}{3(x-1)} - \frac{1}{3(x+2)} - \frac{1}{(x+2)^2}\right] \cdot dx$$

$$= \frac{1}{3} \frac{\log(x-1)}{1} - \frac{1}{3} \frac{\log(x+2)}{1} - \frac{(-1/x+2)}{1}$$

$$= \frac{1}{3}\log(x-1) - \frac{1}{3}\log(x+2) + \frac{1}{x+2} + C$$

Evaluate $\int \frac{2x+1}{(x-1)(x^2+1)} dx$

Given $\int \frac{2x+1}{(x-1)(x^2+1)} \cdot dx$ —— ①

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} \text{ —— ①}$$

$$\frac{2x+1}{\cancel{(x-1)(x^2+1)}} = \frac{A(x^2+1) + (Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$$

$2x+1 = A(x^2+1) + (Bx+C)(x-1) \longrightarrow ②$

To find A,B,C

Put $x=1$, we get

$2(1)+1 = A((1)^2+1) + (B(1)+C)(1-1)$

$2+1 = A(1+1)+(B+C)(0)$

$3 = 2A+0$

$3/2 = A$

$\therefore \boxed{A = 3/2}$

From eq ② To find B,C values for simplify, we get

$2x+1 = A(x^2+1)+(Bx+C)(x-1)$

$2x+1 = Ax^2+A+Bx^2-Bx+Cx-C$

Now compare 'x^2' co-efficient, we get

$0 = A+B$

$0 = \frac{3}{2}+B$

$\therefore \boxed{B = -3/2}$

Now compare 'x' co-efficient, we get

$2 = -B+C$

$2 = -(-3/2)+C$

$2 = 3/2+C$

$2-3/2 = C$

$\frac{4-3}{2} = C$

$\frac{1}{2} = C$

$\therefore \boxed{C = \frac{1}{2}}$

Substitute A,B,C values in eq ① we get

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3/2}{(x-1)} + \frac{-3/2\,x+1/2}{(x^2+1)}$$

$$= \frac{3}{2(x-1)} + \frac{-3x+1}{2(x^2+1)}$$

From eq ① becomes

$$\int\left[\frac{3}{2(x-1)} + \frac{3x+1}{2(x^2+1)}\right]\cdot dx$$

$$= \frac{3}{2}\int\frac{1}{x-1}\cdot dx + \int -\frac{3x}{2(x^2+1)}\cdot dx + \int\frac{1}{2(x^2+1)}\cdot dx$$

$= \frac{3}{2} \frac{\log(x-1)}{1} - \frac{3}{2} \cdot \frac{1}{2} \int \frac{2x}{x^2+1} \cdot dx + \frac{1}{2} \int \frac{1}{1+x^2} dx$

$= \frac{3}{2} \log(x-1) - \frac{3}{2} \cdot \frac{1}{2} \log(x^2+1) + \frac{1}{2} \tan^{-1} x$ $\left[\frac{d}{dx}(x^2+1) = 2x\right.$

$= \frac{3}{2} \log(x-1) - \frac{3}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + c$ //

Integration by parts

Formula $\int dx = x$

* $\int u\,dv = uv - \int v.du$ [log term]

* $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$

where v_1 - integration at 1 time

u' - differentiation at 1 time.

Evaluate $\int \frac{\log x}{u} \cdot \frac{dx}{dv}$

Sol

Given $\int \log x \cdot dx$

We know that

$\int u\,dv = uv - \int v\,du \rightarrow (1)$

$\boxed{u = \log x}$

diff w.r. to x

$\frac{du}{dx} = 1/x$

$\boxed{du = 1/x \cdot dx}$

$dv = dx$

Integration on both sides

$\int dv = \int dx$

$\boxed{v = x}$

From eq (1) becomes

$\int \log x\,dx = \log x \cdot x - \int \cancel{x}\, 1/\cancel{x} \cdot dx$

$\int \log x\,dx = x \log x - x + c$ //

20/01/2020

Evaluate $\int x^2 e^{3x} dx$ (or) $\int e^{3x} x^2 dx$

Given

$$\int x^2 e^{3x} dx$$

We know that

$$\int uv dx = uv_1 - u'v_2 + u''v_3 - \cdots$$

where v_1 = Integration at 1 time

u' = differentiation at 1 time

$$\int x^2 e^{3x} dx = \underset{\text{diff}}{(x^2)}\underset{\text{integration}}{\left(\frac{e^{3x}}{3}\right)} - (2x)\left(\tfrac{1}{3}\frac{e^{3x}}{3}\right) + (2(1))\left(\tfrac{1}{9}\frac{e^{3x}}{3}\right) - 0$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + c$$

1. Evaluate $\int \cos x \cdot x^2 dx$ $\left[x^2 \sin x + 2x\cos x - 2\sin x + c\right]$

2. Evaluate $\int e^{5x} x \cdot dx$ $\left[\frac{1}{5} xe^{5x} - \frac{1}{25} e^{5x} + c\right]$

Sol Given $\int e^{5x} x \cdot dx = \int x \cdot e^{5x} dx$

We know that

$$\int uv dx = uv_1 - u'v_2 + u''v_3 - \cdots$$

Where v_1 = Integration at 1 time

u' = differentiation at 1 time

$$\int \underset{u}{x} \cdot \underset{v}{e^{5x}} dx = (x)(e^{5x}/5) - (1)\left(\tfrac{1}{5}\frac{e^{5x}}{5}\right) + 0$$

$$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + c$$

$$\therefore \int e^{5x} x \cdot dx = \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + c$$

1. Evaluate $\int \cos x \cdot x^2 \cdot dx$

Sol Given

$$\int \cos x \cdot x^2 dx = \int x^2 \cos x \cdot dx$$

We know that

$$\int uv dx = uv_1 - u'v_2 + u''v_3 - \cdots\cdots$$

where

v_1 = Integration at 1 time

u' = differentiation at 1 time

$$\int \underset{v}{\cos x} \cdot \underset{u}{x^2} dx = (x)^2 (\sin x) - (2x)(-\cos x) + (2(1)(-\sin x)) - 0$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + c.$$

$$\therefore \int \cos x \cdot x^2 dx = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Definite integrals

Formula

1. $\sin \pi = \sin 2\pi = \sin 3\pi = \cdots\cdots = 0$
2. $\cos \pi = \cos 3\pi = \cos 5\pi - \cdots\cdots = -1$
3. $\cos 2\pi = \cos 4\pi = \cos 6\pi = \cdots\cdots = 1$

1. Evaluate $\int_0^1 (2x+3)^2 \cdot dx$

Given $\int_0^1 (2x+3)^2 \cdot dx$

$$= \int_0^1 [(2x)^2 + (3)^2 + 2(2x)(3)] \cdot dx \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$$

$$= \int_0^1 [4x^2 + 9 + 12x] \cdot dx$$

$$= \left[4 \frac{x^3}{3} + 9x + 12 \frac{x^2}{2}\right]_0^1$$

In place of x substitute 1 upper limit

Upper limit - lower limit.

$$= \left[4 \frac{(1)^3}{3} + 9(1) + 6(1)^2\right] - [0]$$

$$= 4/3 + 9 + 6$$

$$= 4/3 + 15$$

Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos x} \cdot dx$

Given $\int_0^{\pi/2} \frac{\sin x}{1+\cos x} \cdot dx$ —— ①

Put $\boxed{t = 1+\cos x}$

Differentiation w.r.to x, we get

$\frac{dt}{dx} = 0 + (-\sin x)$

$dt = -\sin x \cdot dx$

$\boxed{-dt = \sin x \cdot dx}$

From eq ① becomes

$\int_0^{\pi/2} \frac{1}{t} (-dt)$

$= \left[-\log t\right]_0^{\pi/2}$

Since $t = 1+\cos x$

$= \left[-\log\left[1+\cos x\right]\right]_0^{\pi/2}$

$= \left[-\log\left(1+\cos \pi/2\right)\right] - \left[-\log\left(1+\cos 0\right)\right]$

$= -\log(1+0) + \log(1+1)$

$= -\log 1 + \log 2 \quad [\because \log 1 = 0]$

$= 0 + \log 2$

$= \log 2$ //

Evaluate (i). $\int_0^{\pi/2} \cos^2 x \cdot dx \quad [\pi/4]$

ii. $\int_0^{\pi/2} \sqrt{1+\sin 2x} \cdot dx \quad [2]$

iii. $\int_0 \sin^3 x \cdot dx$ [ ]

iv. $\int_0^{\pi/4} \cos^4 x \cdot dx$ $\left[\frac{1}{32}(3\pi + 8) \right]$

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$[x = \pi - x]$

Let

$$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} \cdot dx \quad \text{—①}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} \cdot dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+(-\cos x)^2} \cdot dx$$

$\sin(180-\theta) = +\sin\theta$
$\cos(180-\theta) = -\cos\theta$

$$I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} \cdot dx - \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} \cdot dx$$

$$I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx - I \quad [\because \text{eq } ①]$$

$$I + I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} \cdot dx \longrightarrow ②$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} \cdot dx \longrightarrow ②$$

Consider

$$\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} \cdot dx \longrightarrow ③$$

Put $\boxed{t = \cos x}$

diff w.r. to x.

$$\frac{dt}{dx} = -\sin x$$

$$dt = -\sin x\, dx$$

$$\boxed{-dt = \sin x\, dx}$$

$\left[\int \frac{1}{1+x^2} dx = \tan^{-1} x \right]$

From eq ③ becomes

$$\int_0^{\pi} \frac{1}{1+t^2} \cdot -dt$$

$$= -\left[\tan^{-1} t\right]_0^{\pi}$$

Since $t = \cos x$

$$= -\left[\tan^{-1}(\cos x)\right]_0^{\pi}$$

Upper limit − lower limit

$$= \left[-\tan^{-1}(\cos\pi)\right] - \left[-\tan^{-1}(\cos 0)\right]$$

$$= -\tan^{-1}(-1) + \tan^{-1} 1 \qquad \left[\because \tan^{-1}(-\theta) = -\tan^{-1}\theta\right]$$

$$= \tan^{-1} 1 + \tan^{-1} 1$$

$$= 2\tan^{-1} 1$$

$$= 2\,\cancel{\tan^{-1}}\left(\cancel{\tan}\,\pi/4\right) \qquad \left[\tan \pi/4 = 1\right]$$

$$\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = 2\pi/4$$

From eq ② becomes

$$\cancel{2}\mathcal{I} = \pi\left(\cancel{2}\pi/4\right)$$

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx = \pi^2/4. \,//$$

i. Evaluate $\int_0^{\pi/2} \cos^2 x \cdot dx$

Sol Given $\int_0^{\pi/2} \cos^2 x \cdot dx$

$$\int_0^{\pi/2} \frac{1+\cos 2x}{2}\cdot dx$$

$$\int_0^{\pi/2} \frac{1}{2} + \frac{\cos 2x}{2}\cdot dx$$

$$= \left[\frac{1}{2}x + \frac{1}{2}\,\frac{\sin 2x}{2}\right]_0^{\pi/2}$$

= Upper limit − lower limit

$$= \left[\frac{1}{2}(\pi/2) + \frac{1}{2}\,\frac{\sin 2(\pi/2)}{2}\right] - \left[\frac{1}{2}(0) + \frac{1}{2}\,\frac{\sin 2(0)}{2}\right]$$

$$= \frac{1}{2}\left[\pi/2 + \sin 2(\pi/2)\right] - 0$$

$$= \frac{1}{2}\left[\pi/2 + \sin 180^\circ\right]$$

$= \frac{1}{2}\left[\pi/2 + 0\right]$

$= \frac{\pi}{4}$ //

ii. Evaluate $\int_0^{\pi/2} \sqrt{1+\sin 2x}\cdot dx$

Sol Given $\int_0^{\pi/2} \sqrt{1+\sin 2x}\cdot dx$

$= \int_0^{\pi/2} \sqrt{1+2\sin x\cdot\cos x}\cdot dx$ $\quad [\sin^2 x + \cos^2 x = 1]$

$= \int_0^{\pi/2} \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\cdot dx$ $\quad [a^2+b^2+2ab = (a+b)^2]$

$a = \sin x,\ b = \cos x$

$= \int_0^{\pi/2} \sqrt{(\sin x + \cos x)^2}\cdot dx$ $\quad \int \sin x\cdot dx = -\cos x$

$\int \cos x\cdot dx = \sin x$

$= \int_0^{\pi/2} (\sin x + \cos x)\cdot dx$

$= \left[-\cos x + \sin x\right]_0^{\pi/2}$

Upper limit - lower limit

$= \left[-\cos \pi/2 + \sin \pi/2\right] - \left[-\cos(0) + \sin(0)\right]$

$= \left[-\cos 90 + \sin 90\right] - \left[-\cos(0) + \sin(0)\right]$

$= [0+1] - [-1+0]$

$= 1+1$

$= 2$ //

iii. Evaluate $\int_0^{\pi/2} \sin^3 x\cdot dx$

Sol Given $\int_0^{\pi/2} \sin^3 x\cdot dx$

$\int_0^{\pi/2} \frac{1}{4}\left[3\sin x - \sin 3x\right]\cdot dx$

$$= \frac{1}{4}\left[3(-\cos x) + \frac{\cos 3x}{3}\right]_0^{\pi/2}$$

$$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi/2}$$

Upper limit − lower limit

$$= \frac{1}{4}\left[\left[-3\cos(\pi/2) + \frac{\cos 3(\pi/2)}{3}\right] - \left[-3\cos(0) + \frac{\cos 3(0)}{3}\right]\right]$$

$$= \frac{1}{4}\left[\left[-3\cos(90) + \frac{\cos 3(90)}{3}\right] - \left[-3(1) + \frac{1}{3}\right]\right]$$

$$= \frac{1}{4}\left[-3(0) + \frac{0}{3}\right] - \left[-3 + \frac{1}{3}\right]$$

$$= \frac{1}{4}\left[(0) - \left[\frac{-9+1}{3}\right]\right]$$

$$= \frac{1}{4}\left[(0) - \left(\frac{-8}{3}\right)\right]$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{8}^{2}}{3}$$

$$= \frac{2}{3} \,//$$

iv. Evaluate $\int_0^{\pi/4} \cos^4 x \cdot dx$

Given

$$\int_0^{\pi/4} \cos^4 x \cdot dx$$

$$\int_0^{\pi/4} (\cos^2 x)^2 dx$$

$$\int_0^{\pi/4} \left[\frac{1+\cos 2x}{2}\right]^2 \cdot dx$$

$$\int_0^{\pi/4} \frac{(1+\cos 2x)^2}{4} \cdot dx \qquad \left[(a+b)^2 = a^2 + 2ab + b^2\right]$$

$$= \frac{1}{4}\int_0^{\pi/4} \left[(1)^2 + 2(1)(\cos 2x) + \cos^2 2x\right] dx$$

$$= \frac{1}{4}\int_0^{\pi/4} \left[1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right]$$

$$= \frac{1}{4}\left[1x + \cancel{2}\frac{\sin 2x}{\cancel{2}} + \frac{1}{2}\left[1x + \frac{\sin 4x}{4}\right]\right]_0^{\pi/4}$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\right]_0^{\pi/4}$$

Upper limit - Lower limit

$$= \frac{1}{4}\left[\left[\pi/4 + \sin 2(\pi/4) + \frac{1}{2}\left(\pi/4 + \frac{\sin 4(\pi/4)}{4}\right)\right] - \left[0 + \sin 2(0) + \frac{1}{2}\left(0 + \frac{\sin 4(0)}{4}\right)\right]\right]$$

$$= \frac{1}{4}\left[(\pi/4 + \sin(90) + \frac{\pi}{8} + (0)) - (0)\right]$$

$$= \frac{1}{4}\left[\left[\pi/4 + 1 + \frac{\pi}{8} + 0\right] - (0)\right]$$

$$= \frac{1}{4}\left[\frac{2\pi + 8 + \pi}{8}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi + 8}{8}\right]$$

$$= \frac{1}{32}(3\pi + 8)$$

$$\therefore \int_0^{\pi/4} \cos^4 x \cdot dx = \frac{1}{32}(3\pi + 8)$$

20/1/2020

UNIT - V

Differential Equation (DE) (or) Derivative

Definition

Order :- Highest derivative is called order

Ex:- 1. $$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 5y = 0$$

Order = 3

(or)

$$y''' + y'' + 5y = 0$$

(or)

$$y_3 + y_2 + 5y = 0$$

2. $$\frac{d^2y}{dx^2} + 5\frac{d^5y}{dx^5} + 7 = 0$$

order = 5

Degree :- Highest derivative of power is called degree.

Ex:- 1. $$\left[\frac{d^2y}{dx^2}\right]^3 + 2\left(\frac{dy}{dx}\right) = 0$$

Degree = 3

Order = 2

2. $$\left(\frac{d^6y}{dx^6}\right) + \left(\frac{d^2y}{dx^2}\right)^4 + 6y = 0$$

order = 6

Degree = 1

1) Find the order and degree of $\left[\frac{d^2y}{dx^2}\right]^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) = 0$

Sol

$$\left[\frac{d^2y}{dx^2}\right]^3 + \left[\frac{dy}{dx}\right]^2 + \sin\left[\frac{dy}{dx}\right] = 0$$

To find degree, order.

Order :- Highest derivative is called order

Degree :- Highest derivative of power is called Degree.

Order = 2

Degree = 3

2. Find the order and degree of $\left[\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^3\right]^{6/5} = 2y$

Sol
$$\left[\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^3\right]^{6/5} = 2y$$

Multiply powers of 5/6 on both sides, we get

$$\left[\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^3\right]^{\cancel{6}/\cancel{5} \times \cancel{5}/\cancel{6}} = (2y)^{5/6}$$

$$\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^3 = (2y)^{5/6}$$

Order = 2

Degree = 1

Linear Differential Equation (LDE)

General form

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

Then the General solution is

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + C$$

1. Solve $(x \log x)\frac{dy}{dx} + y = 2\log x$

Sol Given

$$(x \log x)\frac{dy}{dx} + y = 2\log x \rightarrow (1)$$

InDirect L.D.E

Now compare

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

Divide $x \log x$ on both sides, we get

$$\frac{x\log x}{x \log x} \cdot \frac{dy}{dx} + \frac{1}{x\log x} \cdot y = \frac{2\log x}{x\log x}$$

$$\frac{dy}{dx} + \frac{1}{x\log x} \cdot y = 2/x$$

It is a L.D.E

Then general solution is

$$ye^{\int P dx} = \int Q e^{\int P dx} dx + c \longrightarrow ②$$

$$P = \frac{1}{x\log x}, \quad Q = 2/x$$

Step-1 :-

$$e^{\int P dx}$$

$$= e^{\int \frac{1}{x\log x} \cdot dx} \longrightarrow ③$$

Consider

$$\int \frac{1}{x\log x} dx \longrightarrow ④$$

Put $\boxed{t = \log x}$

Diff w.r. to x

$$\frac{dt}{dx} = 1/x$$

$$\boxed{dt = \frac{1}{x} \cdot dx}$$

From eq ④ becomes

$$\int \frac{1}{t} \cdot dt$$

$$= \log t$$

Since $t = \log x$

$$\int \frac{1}{x\log x} \cdot dx = \log(\log x)$$

From eq ③ becomes

$$= e^{\log(\log x)}$$

$$e^{\int P dx} = \log x$$

Step-2 :-

$$\int Q e^{\int P dx} \cdot dx$$

$$= \int 2/x \log x \cdot dx$$

$= 2\int t^1 \cdot dt$ $\left[\int x^n \cdot dx = \frac{x^{n+1}}{n+1}\right]$

$= \cancel{2}\, t^2/\cancel{2}$

$\int Qe^{\int P dx} \cdot dx = (\log x)^2$

From eq ② becomes

$y \log x = (\log x)^2 + C$ //

Solve $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$

Sol Given $\frac{dy}{dx}$

22/1/2020 Method-1 Variable seperable Method

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$g(y) \cdot dy = f(x) \cdot dx$$

Integrating.

1. Solve $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$

Sol Given $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$ [v.s]

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$dy = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \cdot dx$$

$$\frac{1}{\sqrt{1-y^2}} \cdot dy = -\frac{1}{\sqrt{1-x^2}} \cdot dx$$

Integrating on both sides

$$\int \frac{1}{\sqrt{1-y^2}} \cdot dy = -\int \frac{1}{\sqrt{1-x^2}} \cdot dx$$

$$\boxed{\sin^{-1} y = -\sin^{-1} x + C}$$

2. solve $(y-yx)\,dx + (x+xy)\,dy = 0$

Sol Given $(y-yx)\,dx + (x+xy)\,dy = 0$

$$y(1-x)\,dx + x(1+y)\,dy = 0 \quad \text{[variable separation]}$$

$$\frac{y(1-x)}{x}\,dx + (1+y)\,dy = 0$$

$$\frac{(1-x)}{x}\,dx + \frac{(1+y)}{y}\,dy = 0 \quad [v.s]$$

Integrating on both sides, we get

$$\int\left(\frac{1-x}{x}\right)dx + \int\left(\frac{1+y}{y}\right) = 0$$

$$\int\left(\frac{1}{x}-\frac{x'}{x}\right)dx+\left(\frac{1}{y}+\frac{y'}{y}\right)dy=0$$

$$=\log x - 1x + \log y + 1y = C$$

(3.) Solve $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

Sol Given

$$\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$$

$$\frac{\sec^2 x}{\tan x}\,dx + \frac{\sec^2 y}{\tan y}\,dy = 0 \quad [\text{V.S}]$$

Integrating on both sides, we get.

$$\int\frac{\sec^2 x}{\tan x}\cdot dx + \int\frac{\sec^2 y}{\tan y}\,dy = C \qquad \frac{\tan x}{\log \sec x}$$

$$\log \tan x + \log \tan y = \log c \qquad \log a + \log b = \log ab$$

$$\cancel{\log}\,[\tan x \cdot \tan y] = \cancel{\log}\, c$$

$$\tan x \cdot \tan y = C$$

(4.) Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Sol Given $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^{-y}\left[e^x + x^2\right]$$

$$dy = e^{-y}\left[e^x + x^2\right] \cdot dx$$

$$\frac{1}{e^{-y}} \cdot dy = \left[e^x + x^2\right]dx \quad [\text{V.S}]$$

Integrating on both sides, we get

$$\int e^y \, dy = \int (e^x + x^2)\cdot dx$$

$$e^y = e^x + \frac{x^3}{3} + C$$

5. Solve $\frac{dy}{dx} = e^{y-x}$ $\quad [-e^{-y} = -e^{-x}+c]$

Given $\frac{dy}{dx} = e^{y-x}$

$$\frac{dy}{dx} = e^{y} \cdot e^{-x}$$

$$dy = (e^{y} \cdot e^{-x})\, dx$$

$$\frac{1}{e^{y}}\, dy = e^{-x} \cdot dx$$

$$e^{-y}\, dy = e^{-x}\, dx$$

Integrating

$$-e^{-y} = -e^{-x}+c$$

6. Solve $\frac{dy}{dx} = (3x+y+4)^2$

Sol Given $\frac{dy}{dx} = (3x+y+4)^2 \longrightarrow ①$

Put $t = 3x+y+4$

Diff w.r to x

$$\frac{dt}{dx} = 3(1) + \frac{dy}{dx} + 0$$

$$\boxed{\frac{dt}{dx} - 3 = \frac{dy}{dx}} \longrightarrow ②$$

Substitute eq ② in eq ①, we get

$$\frac{dt}{dx} - 3 = t^2$$

$$\frac{dt}{dx} = t^2+3$$

$$dt = (t^2+3)\, dx$$

$$\frac{1}{t^2+3}\, dt = dx \quad [V.S]$$

Integrating on both sides, we get

$$\int \frac{1}{t^2+3}\, dt = \int dx \qquad \left[\because \int \frac{1}{x^2+a^2} \cdot dx = \frac{1}{a} \tan^{-1}(x/a)\right]$$

$x = t,\ a = \sqrt{3}$

$$\int \frac{1}{t^2+(\sqrt{3})^2}\, dt = x$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) = x$$

Since $t = 3x+y+4$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{3x+y+4}{\sqrt{3}}\right) = x+c$$

7. Solve $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

Sol Given

$$\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$$

$$\frac{dy}{dx} = -\frac{(y^2+y+1)}{(x^2+x+1)}$$

$$dy = -\frac{(y^2+y+1)}{(x^2+x+1)} \cdot dx$$

$$\frac{1}{y^2+y+1} \cdot dy = -\frac{1}{x^2+x+1} dx \quad [V.S]$$

Integrating on both sides, we get

$$\int \frac{1}{y^2+y+1} dy = -\int \frac{1}{x^2+x+1} dx \longrightarrow (1)$$

Consider

$$\int \frac{1}{x^2+x+1} \cdot dx \qquad = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

from eq (1) becomes

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = -\left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right.$$

Exact Differential Equation

General form $Mdx + Ndy = 0$

Check :- $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Exact D.E

Then the general solution is

$$\underset{y\,\text{constant}}{\int M\,dx} + \underset{\text{donot 'x' term}}{\int N\,dy} = C$$

7. Solve $(x+2y-3)\,dy - (2x-y+1)\,dx = 0$

Sol Given
$(x+2y-3)\,dy - (2x-y+1)\,dx = 0$

Now compare

$$M\,dx + N\,dy = 0$$

$M = -(2x-y+1)$.

Check

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Exact . D.E

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[-2x+y-1]$$

$$= 0+1-0$$

$$\frac{\partial M}{\partial y} = 1$$

$N = (x+2y-3)$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x+2y-3)$$

$$= 1+0-0$$

$$\frac{\partial N}{\partial x} = 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

It is a Exact Differential Equation

Then, the general solution is

$$\int M\,dx + \int N\,dy = C$$

$$\underset{y\,\text{constant}}{\int(-2x+y-1)\,dx} + \underset{\text{Do not }x\text{ term}}{\int(\cancel{x}+2y-3)\cdot dy} = C$$

$$-\frac{\cancel{2}x^2}{\cancel{2}} + yx - 1x + \cancel{2}\frac{y^2}{\cancel{2}} - 3y = C$$

$$-x^2 + yx - x + y^2 - 3y = C$$

Homogeneious Differential Equation

To convert Exact D.E

1. Solve $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$

Sol Given

$$\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$$

It is a homogeneous.

To convert Exact D.E

$$Mdx + Ndy = 0$$

$$(x^2+y^2)dy = 2xy\,dx$$

$$2xy\,dx - (x^2+y^2)dy = 0$$

$$Mdx + Ndy = 0$$

$M = 2xy$ $\quad$ $N = -(x^2+y^2)$

$N = -x^2-y^2$

Check

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow$ E.D.F

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy)$$

$$= 2x(1)$$

$$\frac{\partial M}{\partial y} = 2x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-x^2-y^2)$$

$$= -2x - 0$$

$$\frac{\partial N}{\partial y} = -2x$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

It is not exact differential equation.

2. Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$

[A. $y\sin x + x\sin y + xy = c$]

Laplace Trans form (LT) | Inverse Laplace Trans form

* $\bar{f}(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$ | * $L^{-1}[\bar{f}(s)] = f(t)$

* $L[1] = \frac{1}{s}$ | * $L^{-1}\left[\frac{1}{s}\right] = 1$

* $L[t^n] = \frac{n!}{s^{n+1}}$ ($n! \mapsto$ factor) | * $L^{-1}\left[\frac{1}{s^{n+1}}\right] = \frac{t^n}{(n+1)!}$

* $L[e^{at}] = \frac{1}{s-a}$ | * $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$

* $L[\sin at] = \frac{a}{s^2+a^2}$ | * $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$

* $L[\cos at] = \frac{s}{s^2+a^2}$ | * $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$

* $L[\sinh at] = \frac{a}{s^2-a^2}$ | * $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\sinh at$

* $L[\cosh at] = \frac{s}{s^2-a^2}$ | * $L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$

* $L[\sqrt{t}] = \frac{\sqrt{\pi}}{2s^{3/2}}$ * $L\left[\frac{1}{\sqrt{t}}\right] = \sqrt{\pi/s}$

1. Find the Laplace trans form of $e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t$

(or)

Find $L[e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t]$

Sol To find $L[e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t]$

$= L\{e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t\}$

$= L[e^{2t}] + 4L[t^3] - 2L[\sin 3t] + 3L(\cos 3t)$

$= \frac{1}{s-2} + 4\frac{3!}{s^{3+1}} - 2\frac{3}{s^2+3^2} + 3\frac{s}{s^2+3^2}$

$= \frac{1}{s-2} + 4\frac{6}{s^4} - \frac{6}{s^2+9} + \frac{3s}{s^2+9}$

$= \frac{1}{s-2} + \frac{24}{s^4} - \frac{6}{s^2+9} + \frac{3s}{s^2+9}$

$L[e^{at}] = \frac{1}{s-a}$

$L[t^n] = \frac{n!}{s^{n+1}}$

$L[\sin at] = \frac{a}{s^2+a^2}$

$L[\cos at] = \frac{s}{s^2+a^2}$

$3! = 3\times 2\times 1 = 6$

Find the LT of $\cos^3 2t$

Sol To find $L[\cos^3 2t]$

We know that

$$\left[\because \cos^3 x = \frac{1}{4}[3\cos x + \cos 3x]\right]$$

$$\cos^3 2t = \frac{1}{4}[3\cos 2t + \cos 3(2t)]$$

Take Laplace on both sides, we get

$$L[\cos^3 2t] = \frac{1}{4}[3L\{\cos 2t\} + L\{\cos 6t\}]$$

$$= \frac{1}{4}\left[3\frac{s}{s^2+2^2} + \frac{s}{s^2+6^2}\right] \qquad \left[\because L\{\cos at\} = \frac{s}{s^2+a^2}\right]$$

$$= \frac{1}{4}\left[\frac{3s}{s^2+4} + \frac{s}{s^2+36}\right]$$

Find the L.T of $\cos t \cdot \cos 2t \cos 3t$

Sol To find $L[\cos t \cos 2t \cos 3t]$ $\quad [\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)]$ $A=2t$, $B=3t$

$$L[\cos t \cos 2t \cos 3t] = L\{\cos t \tfrac{1}{2} 2\cos 2t \cos 3t\}$$

$$= \frac{1}{2}L[\cos t[\cos(2t+3t) + \cos(2t-3t)]]$$

$$= \frac{1}{2}L[\cos t(\cos 5t + \cos t)]$$

$$= \frac{1}{2}L[\cos t\cos 5t + \cos^2 t] \qquad \left[\cos^2 x = \frac{1+\cos 2x}{2}\right]$$

$$= \frac{1}{2}L\left[\frac{1}{2}2\cos t\cos 5t + \frac{1+\cos 2t}{2}\right] \qquad [A=t, B=5t]$$

$$\left[\because L\{\cos at\} = \frac{s}{s^2+a^2}\right]$$

$$= \frac{1}{2}\cdot\frac{1}{2}L[\cos(t+5t) + \cos(t-5t) + 1 + \cos 2t]$$

$$= \frac{1}{4}L[\cos 6t + \cos 4t + 1 + \cos 2t]$$

$$= \frac{1}{4}[L\{\cos 6t\} + L\{\cos 4t\} + L\{1\} + L\{\cos 2t\}]$$

$$= \frac{1}{4}\left[\frac{s}{s^2+6^2} + \frac{s}{s^2+4^2} + \frac{1}{s} + \frac{s}{s^2+2^2}\right]$$

$$= \frac{1}{4}\left[\frac{s}{s^2+36} + \frac{s}{s^2+16} + \frac{1}{s} + \frac{s}{s^2+4}\right]$$

Formula :- 1. $\sinh x = \frac{e^{x} - e^{-x}}{2}$

2. $\cosh x = \frac{e^{x} + e^{-x}}{2}$

1. Find the. L.T of $\sinh^3 2t$

Sol To find $L\{\sinh^3 2t\}$.

We know that

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\sinh 2t = \frac{e^{2t} - e^{-2t}}{2}$$

cube on both sides, we get

$$(\sinh 2t)^3 = \left[\frac{e^{2t} - e^{-2t}}{2}\right]^3$$

$$\sinh^3 2t = \frac{(e^{2t} - e^{-2t})^3}{8}$$

Take Laplace on both sides, we get $\left[(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\right]$

$$L\left[\sinh^3 2t\right] = \frac{1}{8} L\left\{(e^{2t} - e^{-2t})^3\right]$$

$$= \frac{1}{8} L\left\{(e^{2t})^3 - 3(e^{2t})^2(e^{-2t}) + 3e^{2t}(e^{-2t})^2 - (e^{-2t})^3\right]$$

$$= \frac{1}{8} L\left\{e^{6t} - 3e^{4t}e^{-2t} + 3e^{2t}e^{-4t} - e^{-6t}\right]$$

$$= \frac{1}{8} L\left\{e^{6t} - 3e^{2t} + 3e^{-2t} - e^{-6t}\right]$$

$$= \frac{1}{8}\left[L\left[e^{6t}\right] - 3L\left\{e^{2t}\right] + 3L\left[e^{-2t}\right] - L\left[e^{-6t}\right]\right]$$

$$= \frac{1}{8}\left[\frac{1}{s-6} - 3\frac{1}{s-2} + 3\frac{1}{s+2} - \frac{1}{s+6}\right] \quad \left[\because L\left[e^{at}\right] = \frac{1}{s-a}\right.$$

$$= \frac{1}{8}\left[\frac{1}{s-6} - \frac{1}{s+6} - 3\left[\frac{1}{s-2} - \frac{1}{s+2}\right]\right] \quad \left. L\left[e^{-at}\right] = \frac{1}{s+a}\right]$$

Inverse L.T Problems

1. Find the inverse L.T of $\frac{2s+3}{s^2+9}$

(or)

Find $L^{-1}\left[\frac{2s+3}{s^2+9}\right]$

Sol

To find $L^{-1}\left[\frac{2s+3}{s^2+9}\right]$

$= L^{-1}\left[\frac{2s}{s^2+3^2}+\frac{3}{s^2+3^2}\right]$

$= 2L^{-1}\left[\frac{s}{s^2+3^2}\right]+3L^{-1}\left[\frac{1}{s^2+3^2}\right]$

$= 2\cos 3t + \cancel{3}\frac{1}{\cancel{3}}\sin 3t$

$= 2\cos 3t + \sin 3t$

$L^{-1}\left[\frac{s}{s^2+a^2}\right]=\cos at$

$L^{-1}\left[\frac{1}{s^2+a^2}\right]=\frac{1}{a}\sin at$

Find $L^{-1}\left[\frac{s+4}{s^3+s}\right]$

Sol To find $L^{-1}\left[\frac{s+4}{s^3+s}\right]$ $[4-4\cos t+\sin t]$

Consider

$\frac{s+4}{s^3+s}=\frac{s+4}{s(s^2+1)}$

$\frac{s+4}{s(s^2+1)}=\frac{A}{s}+\frac{Bx+C}{s^2+1}$

Applications to solve Linear D.E

Formula:- $\ast\ L[y] = \bar{f}(s)$ $\qquad \ast\ y = \bar{f}(0)$

$\ast\ L[y'] = s\bar{f}(s) - \bar{f}(0)$

1. Solve $y' + 5y = e^{-4x}$ given $y(0) = 2$

Sol Given

$y' + 5y = e^{-4x},\ y(0) = 2$

$x = 0,\ y = 2$

Take Laplace on both sides, we get $\qquad \left[\because L[e^{-at}] = \frac{1}{s+a}\right]$

$L[y'] + 5L[y] = L[e^{-4x}]$

$s\bar{f}(s) - \bar{f}(0) + 5\bar{f}(s) = \frac{1}{s+4}$

$\bar{f}(s)[s+5] - \bar{f}(0) = \frac{1}{s+4}$

$\bar{f}(s)[s+5] - y = \frac{1}{s+4}$

$\bar{f}(s)(s+5) - 2 = \frac{1}{s+4}$

$\bar{f}(s)(s+5) = \frac{1}{s+4} + 2$

$= \frac{1 + 2(s+4)}{s+4}$

$= \frac{1 + 2s + 8}{s+4}$

$\bar{f}(s)(s+5) = \frac{2s+9}{s+4}$

$\bar{f}(s) = \frac{2s+9}{(s+4)(s+5)} \qquad \left[\because \bar{f}(s) = L[y]\right]$

$L[y] = \frac{2s+9}{(s+4)(s+5)}$

$y = L^{-1}\left[\frac{2s+9}{(s+4)(s+5)}\right] \longrightarrow ①$

Consider

$\frac{2s+9}{(s+4)(s+5)} = \frac{A}{s+4} + \frac{B}{s+5}$

$$\frac{2s+9}{(s+4)(s+5)} = \frac{1}{s+4} + \frac{1}{s+5}$$

From eq ① becomes

$$y = L^{-1}\left[\frac{1}{s+4} + \frac{1}{s+5}\right]$$

$$= L^{-1}\left[\frac{1}{s+4}\right] + L^{-1}\left[\frac{1}{s+5}\right]$$

$$= e^{-4x} + e^{-5x}$$

6/11/2019

UNIT-3
Differentiation

Basic Formulas

1. $\frac{d}{dx} \cdot x^n = nx^{n-1}$

Ex:- $\frac{d}{dx} \cdot x^5 = 5x^{5-1} = 5x^4$

$\frac{d}{dx} x^7 = 7x^6$

2. $\frac{d}{dx} \cdot x = 1$

3. $\frac{d}{dx} \cdot c = 0$

Ex:- $\frac{d}{dx} \cdot 5 = 0$

4. $\frac{d}{dx}\left(1/x^n\right) = \frac{-n}{x^{n+1}}$

Ex:- $\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{-4}{x^{4+1}}$

5. $\frac{d}{dx} \cdot 1/x = \frac{-1}{x^2}$

6. $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

7. $\frac{d}{dx} \log x = \frac{1}{x}$

8. $\frac{d}{dx} \sin x = \cos x$

9. $\frac{d}{dx} \sin ax = \cos ax \, a$

Ex:- $\frac{d}{dx} \sin 4x = \cos 4x \cdot 4$

10. $\frac{d}{dx} \cos x = -\sin x$

11. $\frac{d}{dx} \cos ax = -\sin ax \cdot a$

Ex:- $\frac{d}{dx} \cos 2x = -\sin 2x \cdot 2$

12. $\frac{d}{dx}\tan x = \sec^2 x$

13. $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

15. $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

16. $\frac{d}{dx}e^x = e^x$

17. $\frac{d}{dx}e^{ax} = e^{ax}\cdot a$

Ex:- 1. $\frac{d}{dx}e^{4x} = e^{4x}\cdot 4$

2. $\frac{d}{dx}e^{-2x} = e^{-2x}\cdot(-2)$

18. $\frac{d}{dx}e^{-x} = -e^{-x}$

19. $\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u$

Ex:- $\frac{d}{dx}\underset{u}{x}\,\underset{v}{e^x} = x\frac{d}{dx}e^x + e^x\frac{d}{dx}x$

$= xe^x + e^x(1)$

20. $\frac{d}{dx}uvw = uv\frac{d}{dx}w + vw\frac{d}{dx}u + wu\frac{d}{dx}v.$

Ex:- $\frac{d}{dx}xe^x\sin x = xe^x\frac{d}{dx}\sin x + e^x\sin x\frac{d}{dx}x + \sin x\, x\frac{d}{dx}e^x$

$= xe^x\cos x + e^x\sin x(1) + \sin x\; xe^x$

21. $\frac{d}{dx}(u/v) = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$

Ex:- $\frac{d}{dx}(x/e^x) = \frac{e^x\,d/dx\,x - x\,d/dx\,e^x}{(e^x)^2}$

$= \frac{e^x(1) - xe^x}{(e^x)^2}$

22. $\frac{d}{dx}\sec x = \sec x\cdot\tan x.$

23. $\frac{d}{dx}a^x = a^x\log a$

Ex:- $\frac{d}{dx}3^x = 3^x\log 3$

1. Find the derivative (or) differentiation of $x^6-6x^5+3x^2+1$

Sol $\frac{d}{dx}(x^6-6x^5+3x^2+1)$

$$\left[\because \frac{d}{dx}x^n = nx^{n-1} \quad \frac{d}{dx}c = 0\right]$$

$= \left[6x^5-6(5x^4)+3(2x^1)+0\right]$

$= 6x^5-30x^4+6x$

2. Find $\frac{dy}{dx}$ if $y = \sqrt{x}-2x+4\sqrt{x^3}$

Sol Given $y = \sqrt{x}-2x+4\sqrt{x^3}$

To find $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}-2x+4\sqrt{x^3})$

$= \frac{d}{dx}\left[\sqrt{x}-2x+4(x^3)^{1/2}\right]$ $\left[\because \frac{d}{dx}x^n = nx^{n-1}\right]$

$= \frac{d}{dx}\left[\sqrt{x}-2x+4(x^{3/2})\right]$

$= \frac{d}{dx}(\sqrt{x}-2x+4x^{3/2})$ $\left[\frac{3-2}{2} = \frac{1}{2}\right]$

$= \frac{1}{2\sqrt{x}}-2(1)+4(3/2\, x^{3/2-1})$

$= \frac{1}{2\sqrt{x}}-2+6x^{1/2}$

3. Differentiation with respect to x: $e^x \tan x$

Sol $\frac{d}{dx}(e^x \tan x)$

$\left[\because \frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$= e^x \, d/dx \tan x + \tan x \frac{d}{dx}e^x$

$= e^x \sec^2 x + \tan x \, e^x$

$\therefore \frac{d}{dx} e^x \tan x = e^x \sec^2 x + \tan x \, e^x$ //

1. Find $\frac{dy}{dx}$ (i) $y = 3\cos x + 2\log x + 21x^2 + 5$

(ii) $y = 3e^x + 5\sin x$

(iii) $x^3 \tan x$

(iv) $(1-x^2)\tan x$

11/11/2019

Differentiation (i) $x \sin x$

(ii) $\sin x \log x$

(iii) $e^{8x} \cdot \sec x$

(B) Differentiation with respect to x : $\frac{e^x + \sin x}{e^x - \cos x}$

Sol

Given

$$\frac{d}{dx}\left[\frac{(e^x + \sin x)}{(e^x - \cos x)}\right]$$

We know that

$$\frac{d}{dx}(u/v) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$\frac{d}{dx}\left(\frac{e^x + \sin x}{e^x - \cos x}\right) = \frac{(e^x - \cos x)\frac{d}{dx}(e^x + \sin x) - (e^x + \sin x)\frac{d}{dx}(e^x - \cos x)}{(e^x - \cos x)^2}$$

$$\left[\because \frac{d}{dx} e^x = e^x , \ \frac{d}{dx} \sin x = \cos x\right]$$

$$= \frac{(e^x - \cos x)(e^x + \cos x) - (e^x + \sin x)(e^x + \sin x)}{(e^x - \cos x)^2}$$

$$= \frac{(e^x)^2 - (\cos x)^2 - [(e^x)^2 + e^x \sin x + e^x \sin x + \sin^2 x]}{(e^x - \cos x)^2} \quad [a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{\cancel{e^{2x}} - \cos^2 - \cancel{e^{2x}} - 2e^x \sin x - \sin^2 x}{(e^x - \cos x)^2}$$

$$= \frac{-(\sin^2 x + \cos^2 x + 2e^x \sin x)}{(e^x - \cos x)^2} \quad \because [\sin^2 x + \cos^2 x = 1]$$

$$= \frac{-(1 + 2e^x \sin x)}{(e^x - \cos x)^2} \;//$$

Differentiation with respect to i. $\frac{\log x}{2 + \log x}$ ii. $\frac{4x+5}{7x-3}$

(iii) $\frac{a - b\cos x}{a + b\cos x}$

Find the derivation of $\frac{x^{10} + 3x^5 + 15x - 1}{x^2}$

Sol

$$\frac{d}{dx}\left[\frac{x^{10} + 3x^5 + 15x - 1}{x^2}\right]$$

$$= \frac{d}{dx}\left[\frac{x^{\cancel{10}\,8}}{\cancel{x^2}} + \frac{3x^{\cancel{5}\,3}}{\cancel{x^2}} + \frac{15\cancel{x}}{x^{\cancel{2}}} - \frac{1}{x^2}\right]$$

$$= \frac{d}{dx}\left[x^8 + 3x^3 + 15/x - \frac{1}{x^2}\right] \quad \left[\because \frac{d}{dx}x^n = nx^{n-1}\right.$$

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n}{x^{n+1}}$$

$$= 8x^7 + 3(3x^2) + 15\left(-\frac{1}{x^2}\right) - \left(\frac{-2}{x^{2+1}}\right) \quad \left.\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}\right]$$

$$= 8x^7 + 9x^2 - 15/x^2 + \frac{2}{x^3}$$

ii. $y = 3e^x + 5\sin x$

Sol

$$\frac{d}{dx}y = 3e^x + 5\sin x \quad \left[\because \frac{d}{dx}\sin x = \cos x\right.$$

$$= 3e^x + 5\cos x \quad \left.\frac{d}{dx}e^x = e^x\right]$$

$$= 3e^x + 5\cos x$$

iii. $x^3 \tan x$

Sol

$$\frac{d}{dx}y = x^3 \tan x$$

$$= \frac{d}{dx} uv = u \frac{d}{dx} v + v \frac{d}{dx} u \qquad \left[\frac{d}{dx} x^n = nx^{n-1}\right]$$

$$= x^3 \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \cdot x^3$$

$$= x^3 \cdot \sec^2 x + \tan x \cdot 3x^2$$

i. $y = 3\cos x + 2\log x + 21x^2 + 5$

$$\frac{d}{dx} y = 3\cos x + 2\log x + 21x^2 + 5$$

$$3(-\sin x) + 2(1/x) + 21(2x) + 0$$

$$\frac{d}{dx} = -3\sin x + 2/x + 42x$$

i. $\dfrac{\log x}{2+\log x}$

$$\frac{d}{dx} \frac{\log x}{2+\log x}$$

$$\therefore \frac{d}{dx} u/v = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$= \frac{2+\log x \frac{d}{dx} \log x - \log x \frac{d}{dx} 2+\log x}{(2+\log x)^2}$$

$$= \frac{2+\log x (1/x) - (\log x)(1/x)}{(2+\log x)^2}$$

$$= \frac{2/x + \frac{\log x}{x} - \frac{\log x}{x}}{(2+\log x)^2}$$

$$= \frac{2/x + \cancel{\log} - \cancel{\log}}{(2+\log x)^2}$$

$$= \frac{2}{x(2+\log x)^2}$$

ii. $\dfrac{4x+5}{7x-3}$

Sol $\dfrac{d}{dx}\dfrac{4x+5}{7x-3}$

$$\because \frac{d}{dx}\,{}^{u}/_{v} = \frac{v\,{}^{d}/_{dx}u - u\,{}^{d}/_{dx}v}{v^2}$$

$$= \frac{(7x-3)\,{}^{d}/_{dx}4x+5 - 4x+5\,\frac{d}{dx}\,7x-3}{(7x-3)^2}$$

$$= \frac{(7x-3)\,{}^{d}/_{dx}(4x+5) - 4x+5\,\frac{d}{dx}(7x-3)}{(7x-3)^2}$$

$7x-3\ (4(1)+0) - 4x+5\ (7(1)-0)$

$$= \frac{(7x-3)(4)-(4x+5)(7)}{(7x-3)^2}$$

$$= \frac{\cancel{28x}-12-\cancel{28x}+35}{(7x-3)^2}$$

$$= \frac{-12-35}{(7x-3)^2}$$

$$= \frac{-47}{(7x-3)^2}$$

iii. $\dfrac{a-b\cos x}{a+b\cos x}$

Sol $\left[\dfrac{d}{dx}\dfrac{a-b\cos x}{a+b\cos x}\right]$

$$\because \frac{d}{dx}\,{}^{u}/_{v} = \frac{v\,{}^{d}/_{dx}\cdot u - u\,{}^{d}/_{dx}\cdot v}{v^2}$$

$$= \frac{(a+b\cos x\,{}^{d}/_{dx}\,a-b\cos x) - (a-b\cos x\,{}^{d}/_{dx}\,a+b\cos x)}{(a+b\cos x)^2}$$

$$= \frac{a+b\cos x(\sin bx b) - (a-b\cos x(\sin bx b)}{(a+b\cos x)^2}$$

$$= \frac{a+b\cos x(\sin x \, b) - (a-b\cos x)(-\sin x \, b)}{(a+b\cos x)^2}$$

wrong

$$= \frac{(\sin x \, b)^2}{(a+b\cos x)^2}$$ //

$$\frac{d}{dx} \frac{a-b\cos x}{a+b\cos x}$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$$

$\frac{d}{dx}\cos x = -\sin x$

$d/dx \, x = 1$

$$= \frac{(a+b\cos x)\frac{d}{dx}(a-b\cos x) - (a-b\cos x)\frac{d}{dx}(a+b\cos x)}{(a+b\cos x)^2}$$

$$= \frac{(a+b\cos x)(+b\sin x) - (a-b\cos x)(-b\sin x)}{(a+b\cos x)^2}$$

$$= \frac{b\sin x((a+\cancel{b\cos x})) + (a-\cancel{b\cos x})\,b\sin x}{(a+b\cos x)^2}$$

$$= \frac{2ab\sin x}{(a+b\cos x)^2}$$ //

i. $x \sin x$ differentiation.

Sol $\frac{d}{dx} \underset{u}{x} \underset{v}{\sin x}$ $u = x, v = \sin x$

$= x\frac{d}{dx}\sin x + \sin x \frac{d}{dx} x$ $\left[\frac{d}{dx} uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$= x\cos x + \sin x (1)$ $\left[\frac{d}{dx}\sin x = \cos x, \; \frac{d}{dx} x = 1\right]$

$= x\cos x + \sin x$ //

ii. $\sin x \log x$ differentiation

Sol $\frac{d}{dx} \underset{u}{\sin x} \underset{v}{\log x}$ $u = \sin x$, $v = \log x$

$= \sin x \frac{d}{dx}\log x + \log x \frac{d}{dx}\sin x \left[\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$= \sin x \cdot \frac{1}{x} + \log x \cdot \cos x \quad \left[\frac{d}{dx}\log x = \frac{1}{x}\right.$

$= \sin x \cdot \frac{1}{x} + \log x \cdot \cos x \;// \quad \left.\frac{d}{dx}\sin x = \cos x\right]$

(B)

iii. $e^{8x} \cdot \sec x$ differentiation.

Sol $\frac{d}{dx} \underset{u}{e^{8x}} \cdot \underset{v}{\sec x} \quad u = e^{8x}, v = \sec x \quad \left[\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$= e^{8x}\, d/dx \sec x + \sec x \frac{d}{dx} e^{8x} \quad \left[\frac{d}{dx}e^{ax} = e^{ax}\cdot a\right.$

$= e^{8x}\, d/dx \sec x + \sec x \cdot e^{8x} \cdot 8 \quad \left.\frac{d}{dx}\sec x = \sec x \cdot \tan x\right]$

$= e^{8x} \sec x \cdot \tan x + \sec x \cdot e^{8x} \cdot 8$

(B)

iv. Find $\frac{dy}{dx}$ if $y = (1-x^2)\tan x$

Sol Given $\therefore y = (1-x^2)\tan x$

$\frac{dy}{dx}$

$\frac{d}{dx} y = \underset{u}{(1-x^2)} \underset{v}{\tan x} \quad u = 1-x^2, \; v = \tan x$

$\left[\text{Formula } \frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$= (1-x^2)\frac{d}{dx}\tan x + \tan x \frac{d}{dx}(1-x^2)$

$= (1-x^2)\sec^2 x + \tan x\,[(0) - 2x]$

$= (1-x^2)\sec^2 x - 2x\tan x. \quad \left[\frac{d}{dx}\tan x = \sec^2 x\right.$

$= (1-x^2)\sec^2 x - 2x\tan x \quad \frac{d}{dx}c = 0$

$= (1-x^2)\sec^2 x - 2x\tan x \;// \quad \left.\frac{d}{dx}x^n = nx^{n-1}\right]$

12/11/2017 Method - II

Derivation of a function of a function (chain Rule).

1. Differentiation with respect to x: e^{4x^2}

Sol

$\frac{d}{dx} \cdot e^{4x^2}$ $\quad \because \left[\frac{d}{dx} e^x = e^x\right]$

$= e^{4x^2} \frac{d}{dx}(4x^2)$ $\quad \frac{d}{dx} x^n = nx^{n-1}$

$= e^{4x^2} \cdot 4(2x)$

$= 8x e^{4x^2}$ //

2. Differentiation with respect to x: $(2x+3)^{10}$

Sol

$\frac{d}{dx}(2x+3)^{10}$ $\quad \left[\because \frac{d}{dx} x^n = nx^{n-1}\right]$

$= 10(2x+3)^{10-1} \, d/dx \, (2x+3)$

$= 10(2x+3)^9 [2(1)+0]$

$= 20(2x+3)^9$ //

3. Differentiation with respect to x. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{10}$

Sol

$\frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{10}$ $\quad \left[\because \frac{d}{dx} x^n = nx^{n-1}\right]$

$= 10\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^9 \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ $\quad \left[\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}\right.$

$= 10\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^9 d/dx \left[\sqrt{x} + \frac{1}{x^{1/2}}\right]$ $\quad \left.\frac{d}{dx}(1/x^n) = \frac{-n}{x^{n+1}}\right]$

$= 10\left(\sqrt{x} + 1/\sqrt{x}\right)^9 \left[\frac{1}{2\sqrt{x}} + \frac{-1/2}{x^{1/2+1}}\right]$ $\quad \left[1/2 + 1 = \frac{1+2}{2} = \frac{3}{2}\right]$

$= 10\left(\sqrt{x} + 1/\sqrt{x}\right)^9 \left[\frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}\right]$ //

4. Differentiation with respect to x: $\sin e^x$

(or)

Find $\frac{dy}{dx}$, if $y = \sin e^x$

Sol Given,

$$y = \sin e^x$$

To find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin e^x) \qquad \left[\because \frac{d}{dx}\sin x = \cos x\right]$$

$$\left[\frac{d}{dx} e^x = e^x\right]$$

$$= \cos e^x \frac{d}{dx} e^x$$

$$= \cos e^x (e^x)$$

Find $\frac{dy}{dx}$, if $y = e^{\sqrt{\sin x}}$

Sol Given,

$$y = e^{\sqrt{\sin x}}$$

To find $\frac{dy}{dx}$ $\qquad \left[\frac{d}{dx} e^x = e^x\right]$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\sqrt{\sin x}}$$

$$= e^{\sqrt{\sin x}} \cdot \frac{d}{dx}\sqrt{\sin x} \qquad \left[\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}\right]$$

$$= e^{\sqrt{\sin x}} \cdot \frac{1}{2\sqrt{\sin x}} \cdot \frac{d}{dx}\sin x \qquad \left[\frac{d}{dx}\sin x = \cos x\right]$$

$$= e^{\sqrt{\sin x}} \cdot \frac{1}{2\sqrt{\sin x}} \cdot \cos x$$

If $y = \log\sqrt{\tan x}$ find $\frac{dy}{dx}$

(B) Find the derivative of $\sin^5 3x \cos^3 2x$

Sol

$$\frac{d}{dx}\left(\underset{u}{\sin^5 3x}\ \underset{v}{\cos^3 2x}\right)$$

$$\left[\frac{d}{dx} uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$$

$= \sin^5 3x \, \frac{d}{dx} \cos^3 2x + \cos^3 2x \, \frac{d}{dx} \sin^5 3x$

$= \sin^5 3x \, \frac{d}{dx} (\cos 2x)^3 + \cos^3 2x \, \frac{d}{dx} (\sin 3x)^5$ $\left[\cos^3 3x = (\ldots\right.$ $\left[\frac{d}{dx} x^n = n x^{n-1}\right.$

$= \sin^5 3x \; 3(\cos 2x)^2 (-\sin 2x \cdot 2) + \cos^3 2x \left[5(\sin 3x)^4 \cos 3x \cdot 3\right]$

$= -6 \sin^5 3x \cos^2 2x \sin 2x + 15 \cos^3 2x \sin^4 3x \cos 3x.$

Find $\frac{dy}{dx}$ if $y = \cos^4 x$

Find the derivative of $\sin^4 x \cos^3 x$

u v method

Sol Let $y = \sin^4 x \cdot \cos^3 x$

$y = (\sin x)^4 \cdot (\cos x)^3$

$\therefore \frac{dy}{dx} = (\sin x)^4 \cdot \frac{d}{dx} (\cos x)^3 + \cos^3 x \frac{d}{dx} (\sin x)^4$

$= (\sin x)^4 \cdot 3(\cos x)^2 \cdot \frac{d}{dx} (\cos x) + (\cos x)^3 \cdot 4(\sin x)^3 \cdot \frac{d}{dx} (\sin x)$

$= 3(\sin x)^4 (\cos x)^2 \cdot (-\sin x) + 4(\cos x)^3 \cdot (\sin x)^3 \cdot (\cos x)$

$= (\sin^3 x)(\cos^2 x)\left[-3\sin^2 x + 4 \cdot \cos^2 x\right]$

(B) If $y = \log\sqrt{\tan x}$ find $\frac{dy}{dx}$

Sol Given

$y = \log\sqrt{\tan x}$

To find $\frac{dy}{dx}$

$\left[\frac{d}{dx} \log x = \frac{1}{x}\right.$

$\left.\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}\right]$

$\frac{dy}{dx} = \frac{d}{dx} \log\sqrt{\tan x}$

$= \frac{1}{\sqrt{\tan x}} \cdot \frac{d}{dx} \sqrt{\tan x}$

$= \frac{1}{\sqrt{\tan x}} \cdot \frac{1}{2\sqrt{\tan x}} \cdot \frac{d}{dx} \tan x$

$= \frac{1}{\sqrt{\tan x}} \cdot \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$

$$= \frac{\sec^2 x}{2(\sqrt{\tan x})^{\cancel{2}}} = \frac{\sec^2 x}{2\tan x}$$

$$\left[\begin{array}{l} \tan x = \frac{\sin x}{\cos x} \\ \sec x = \frac{1}{\cos x} \\ \operatorname{cosec} x = \frac{1}{\sin x} \end{array}\right]$$

$$= \frac{1/\cos^{\cancel{2}} x}{2\,\frac{\sin x}{\cancel{\cos x}}} = \frac{1}{2\cos x \sin x} \,//$$

(or)

$$= \frac{1}{\sin 2x} = \operatorname{cosec} 2x$$

Find $\frac{dy}{dx}$ if $y = \cos^4 x$

Sol Given $y = \cos^4 x$

$$\frac{dy}{dx} = \frac{d}{dx}\cos^4 x \qquad \left[\cos^4 x = (\cos x)^4\right]$$

$$= \frac{d}{dx}(\cos x)^4 \qquad \left[\frac{d}{dx}x^n = nx^{n-1}\right]$$

$$= 4(\cos x)^3 \cdot d/dx \cos x$$

$$= -4(\cos x)^3 \cdot \sin x \,//$$

(or)

$$= 4\cos^3 x \cdot \sin x \,//$$

2/11/2019 **Logarithemic differentiation**

1. Find the differentiation of x^x.

Sol Let $y = x^x$

Take 'log' on both sides, we get

$$\log y = \log x^x \qquad \left[\because \log a^b = b\log a\right]$$

$$\log y = \underset{u}{x}\,\underset{v}{\log x} \qquad \left[\log x = \frac{1}{x}\right]$$

Differentiate with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log x + \log x\frac{d}{dx}x \qquad \left[\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \cancel{x}\frac{1}{\cancel{x}} + \log x\,(1)$$

$$\frac{dy}{dx} = y\left[1 + \log x\right]$$

Since $y = x^x$

$$\frac{dy}{dx} = x^x [1 + \log x]$$

(2) Find $\frac{dy}{dx}$ if $y = \sin x^{\tan x}$

Sol (B)

~~Let~~ Given $y = \sin x^{\tan x}$

Take 'log' on both sides

$$\log y = \log \sin x^{\tan x} \qquad [\log a^b = b \log a]$$

$$\log y = \underset{u}{\tan x} \; \underset{v}{\log (\sin x)} \qquad \left[\log x = \frac{1}{x}\right]$$

Differentiate with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \tan x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \tan x$$

$$\frac{1}{y}\frac{dy}{dx} = \tan x \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot \sec^2 x \qquad \left[\frac{d}{dx} uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$$

$$\frac{dy}{dx} = y\left[\tan x \cdot \left[\frac{1}{\sin x} \cdot \cos x\right] + \log \sin x \cdot \sec^2 x\right]$$

$$\frac{dy}{dx} = y\left[(\tan x)(\cot x)\right] + \log \sin x \cdot \sec^2 x$$

$$\frac{dy}{dx} = y\left[1 + \log \sin x \cdot \sec^2 x\right]$$

We know that

$$y = (\sin x)^{\tan x}$$

$$\therefore \frac{dy}{dx} = (\sin x)^{\tan x}\left[1 + \log \sin x \cdot \sec^2 x\right]$$

$$\therefore \frac{dy}{dx} = (\sin x)^{\tan x}\left[1 + \log \sin x \cdot \sec^2 x\right]$$

Maxima (or) Minima

Working Method

Step-1: $\left[\frac{dy}{dx}\right] = 0 \Rightarrow$ To find point

Step-2:

* $\frac{d^2y}{dx^2} \overset{\text{less than}}{<} 0 \Rightarrow$ then the function is maximum.

* $\frac{d^2y}{dx^2} \overset{\text{greater}}{>} 0 \Rightarrow$ then the function is minimum.

* $\frac{d^2y}{dx^2} = 0 \Rightarrow$ then the function is neither maximum (or) minimum.

Find the maximum and minimum values of

(B) $x^4+2x^3-3x^2-4x+4$

Sol Let $y = x^4+2x^3-3x^2-4x+4 \longrightarrow ①$

Differentiation with respect to x in eq ①, we get

$$\frac{dy}{dx} = 4x^3+\underline{2(3x^2)} - 3(2x) - 4(1) + 0$$

$$\frac{dy}{dx} = 4x^3+6x^2-6x-4 \longrightarrow ②$$

Assume $\frac{dy}{dx} = 0$

$\Rightarrow 4x^3+6x^2-6x-4 = 0$

$x=1$	4	6	-6	-4
		4	10	4
	4	10	4	0

$4x^2+10x+4 = 0$

$2(2x^2+5x+2) = 0$

$2x^2+5x+2 = 0$

$2x^2+4x+1x+2 = 0$

4: 4, 1

Two number multiply = 4

Two number add = 5

$2x(x+2)+1(x+2) = 0$

$(x+2)(2x+1) = 0$

$x+2=0$	$2x+1=0$
$x=-2$	$2x=-1$
	$x=-1/2$

$\therefore x=1, x=-2, x=-1/2$

Step-1: at $x=1$

Diff with respect to x in eq ②, we get

$$\frac{d^2y}{dx^2} = 4(3x^2)+6(2x)-6(1)-0$$

$$\frac{d^2y}{dx^2} = 12x^2+12x-6$$

at $x=1$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 12(1)^2+12(1)-6 = 12+12-6 = 18.$$

$\frac{d^2y}{dx^2} > 0$ then the function is minimum in eq ①, we have

$y = (1)^4 + 2(1)^3 - 3(1)^2 - 4(1) + 4$

$= 1 + 2 - 3 - \cancel{4} + \cancel{4}$

$= 0$.

$\therefore$ The minimum value is 0.

Step - II at $x = -2$

$\left[\frac{d^2y}{dx^2}\right]_{x=-2} = 12x^2 + 12x - 6$

$= 12(-2)^2 + 12(-2) - 6$

$= 12(4) - 24 - 6$

$= 48 - 30 = 18$

$\frac{d^2y}{dx^2} > 0$ then the function is ~~maximum~~ minimum

$x = -2$ substitute in eq ①, we have

$y = x^4 + 2x^3 - 3x^2 - 4x + 4$

$= (-2)^4 + 2(-2)^3 - 3(-2)^2 - 4(-2) + 4$

$= 16 + 2(-8) - 3(4) + 8 + 4$

$= 16 - 16 - 12 + 12 = 0$

$\therefore$ The minimum value is 0.

Step - III

at $x = -1/2$.

$\left[\frac{d^2y}{dx^2}\right]_{x=-1/2} = 12x^2 + 12x - 6$

$= 12(-1/2)^2 + 12(-1/2) - 6$

$= 12(1/4) - 6 - 6 = 3 - 12 = -9$

$\frac{d^2y}{dx^2} < 0$ then the function is maximum

$x = -1/2$ substitute in eq ①, we have

$y = x^4 + 2x^3 - 3x^2 - 4x + 4$

$= (-1/2)^4 + 2(-1/2)^3 - 3(-1/2)^2 - 4(-1/2) + 4$

$$= \frac{1}{16} + \cancel{2}\left(-\frac{1}{\cancel{8}_{4}}\right) - 3\left(\frac{1}{4}\right) + \frac{\cancel{4}^{2}}{\cancel{2}} + 4$$

$$= \frac{1}{16} - \frac{1}{4} - \frac{3}{4} + \frac{2+4}{1} \Rightarrow \frac{1}{16} - \frac{1}{4} - \frac{3}{4} + 6$$

$$= \frac{1-4-12+96}{16}$$

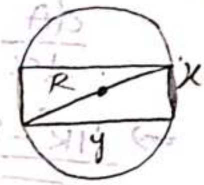
$$= \frac{97-16}{16} = \frac{81}{16}$$

$\therefore$ The maximum value is $\frac{81}{16}$.

4/11/2019

(B) The maximum rectangle that can be described is a circle is a square.

Sol Let x be a length and 'y' be the breadth of a rectangle.



Let 'R' be the radius of the circle

We know that equation of the circle is

$$x^2+y^2=r^2$$

By data $x^2+y^2=(2R)^2$

$$\Rightarrow y^2 = 4R^2 - x^2$$

$$\Rightarrow y = \sqrt{4R^2-x^2} \quad \text{——} \quad (1)$$

Let 'A' be the area of the rectangle is

$$A = \text{length} \times \text{breadth}$$

$$A = xy$$

$$A = x\sqrt{4R^2-x^2}$$

Differentiation w.r. to x, we get

Formulas
$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}, \quad \frac{d}{dx}x = 1, \quad \frac{d}{dx}c = 0, \quad \frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{dA}{dx} = x\frac{d}{dx}\sqrt{4R^2-x^2} + \sqrt{4R^2-x^2}\,\frac{d}{dx}x$$

$$= x\frac{1}{2\sqrt{4R^2-x^2}}\cdot\frac{d}{dx}(4R^2-x^2) + \sqrt{4R^2-x^2}\,(1)$$

$$= x\frac{1}{\cancel{2}\sqrt{4R^2-x^2}}\cdot(0-\cancel{2}x) + \sqrt{4R^2-x^2}$$

$$= \frac{-x^2}{\sqrt{4R^2-x^2}} + \sqrt{4R^2-x^2}$$

$$= \frac{-x^2 + \left(\sqrt{4R^2-x^2}\right)^2}{\sqrt{4R^2-x^2}}$$

$$= \frac{-x^2 + 4R^2 - x^2}{\sqrt{4R^2-x^2}}$$

$$\frac{dA}{dx} = \frac{4R^2 - 2x^2}{\sqrt{4R^2-x^2}}$$

By data the maximum rectangle, we have

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \frac{4R^2-2x^2}{\sqrt{4R^2-x^2}} = 0$$

$$\Rightarrow \frac{4R^2-2x^2}{1} = 0 \times \sqrt{4R^2-x^2}$$

$$\Rightarrow 4R^2 - 2x^2 = 0$$

$$\Rightarrow 4R^2 = 2x^2$$

$$\therefore \cancel{2}x^2 = \cancel{4}^{2}R^2$$

$$x^2 = 2R^2$$

$$x = \sqrt{2R^2} = \sqrt{2}\,R$$

Substitute 'x' value in eq ①, we get

$$y = \sqrt{4R^2 - x^2}$$

$$= \sqrt{4R^2 - (\sqrt{2}R)^2}$$

$$= \sqrt{4R^2 - (\sqrt{2})^2(R^2)}$$

$$= \sqrt{4R^2 - 2R^2}$$

$$= \sqrt{2R^2}$$

$$y = \sqrt{2}\,R$$

$\therefore$ Length $x = \sqrt{2}\,R$

Breadth $y = \sqrt{2}\,R$

$\therefore$ The rectangle is square.

Successive Differentiation

* $\frac{dy}{dx} = y_1 = y'$

* $\frac{d^2y}{dx^2} = y_2 = y''$

1. If $y = \frac{6+5x}{2+3x}$ find $\frac{d^2y}{dx^2}$

Sol Given

$$y = \frac{6+5x}{2+3x} \longrightarrow (1)$$

To find $\frac{d^2y}{dx^2}$

Differentiation w.r to in eq (1), we get

$$\frac{dy}{dx} = \frac{(2+3x)\frac{d}{dx}(6+5x) - (6+5x)\, d/_{dx}(2+3x)}{(2+3x)^2} \quad \left[\frac{d}{dx} u/v = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}\right]$$

$$= \frac{(2+3x)(5) - (6+5x)(3)}{(2+3x)^2} \quad \left[\frac{d}{dx}\cdot c = 0 \quad \frac{d}{dx}x = 1\right]$$

$$= \frac{10 + \cancel{15x} - 18 - \cancel{15x}}{(2+3x)^2}$$

$$\frac{dy}{dx} = \frac{-8}{(2+3x)^2}$$

Again differentiate w.r. to x, we get

$$\frac{d^2y}{dx^2} = -8\left[\frac{-2}{(2+3x)^{2+1}}\right]\frac{d}{dx}(2+3x) \quad \left[\frac{d}{dx}\,1/x^n = \frac{-n}{x^{n+1}}\right]$$

$$= \frac{16}{(2+3x)^3}(3) = \frac{48}{(2+3x)^3} \;// \quad \left[\frac{d}{dx}c = 0 \quad \frac{d}{dx}x = 1\right]$$

18/11/2019 If $y = \cos(\log x)$ show that $x^2 y_2 + xy' + y = 0$

Sol Given

$y = \cos(\log x) \rightarrow (1)$

To show that $x^2 y_2 + xy' + y = 0$ $\left[\because y_2 = \frac{d^2y}{dx^2}, y' = \frac{dy}{dx}\right]$

Diff w.r. to x in eq (1), we get

$\frac{dy}{dx} = -\sin(\log x) \frac{d}{dx}\log x$ $\left[\frac{d}{dx}\cos x = -\sin x\right]$

$y' = -\sin(\log x) \cdot 1/x$

$xy' = -\sin(\log x)$ $\leftarrow (1)$ $\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u$

Again diff w.r. to x, we get

$x\frac{d}{dx}y' + y'\frac{d}{dx}x = -\cos(\log x) 1/x$

$\Rightarrow x[xy'' + y'(1)] = -y$ [$\because$ From eq (1)]

$x^2y'' + xy' + y = 0$ (or)

$x^2 y_2 + xy_1 + y = 0$.

2. If $y = \sqrt{\sec 2x}$, then show that $y_2 = 3y^5 - y$

Sol Given

$y = \sqrt{\sec 2x} \rightarrow (1)$

To show that $y_2 = 3y^5 - y$

From eq (1) becomes

Squaring on both sides, we get

$y^2 = \sec 2x \rightarrow (2)$

Diff w.r. to x, we get

$2yy' = \sec 2x \tan 2x \frac{d}{dx}(2x)$ $\left[\frac{d}{dx}\sec x = \sec x \cdot \tan x\right]$

$\cancel{2}\cancel{y}y' = y^{\cancel{2}} \tan 2x \cdot (\cancel{2})$

$y' = y \tan 2x \rightarrow (3)$

Again diff w.r. to x $\left[\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u\right]$

$y'' = y\sec^2 2x \frac{d}{dx}2x + \tan 2x y'$ $\left(\frac{d}{dx}\tan x = \sec^2 x\right)$

$y'' = y\sec^2 2x(2) + \tan 2x\, y \tan 2x$ [$\because$ eq (3)]

$$y'' = 2y\sec^2 2x + y\tan^2 2x$$
$$= 2y(y^2)^2 + y[\sec^2 2x - 1]$$
$$= 2y^5 + y[(y^2)^2 - 1]$$

$[\sec^2\theta - \tan^2\theta = 1$
$\tan^2\theta = \sec^2\theta - 1]$

$[\because eq ②]$

$$y'' = 2y^5 + y^5 - y$$
$$y'' = 3y^5 - y$$
(or)
$$y_2 = 3y^5 - y \quad //$$

9/11/2019

(B) 1. If $x = t^2$, $y = t^3$ then find y_2

sol Given
$$x = t^2,\ y = t^3$$
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \longrightarrow ①$$

Since $x = t^2$
diff. w.r.to t
$$\frac{dx}{dt} = 2t$$

since $y = t^3$
diff w.r.to t
$$\frac{dy}{dt} = 3t^2$$

from eq ① becomes
$$\frac{dy}{dx} = \frac{3t^{\cancel{2}}}{2\cancel{t}}$$
$$\therefore \frac{dy}{dx} = \frac{3}{2}t \text{ ——— } ②$$

To find y_2 (or) $\frac{d^2y}{dx^2}$

Again differentiation w.r.to x, we get
$$\frac{d^2y}{dx^2} = \frac{3}{2}(1)\frac{dt}{dx}$$
$$= \frac{3}{2}\frac{1}{dx/dt} \quad (\because \text{from eq } ②)$$
$$= \frac{3}{2}\frac{1}{2t} \quad \left[\frac{dt}{dx} = 1/dx/dt\right]$$
$$= \frac{3}{4t} \quad //$$

Differentiation of functions from the first principles

$$\frac{dy}{dx} = \underset{h\to 0}{Lt} \frac{f(x+h) - f(x)}{h}$$

1. Find the derivative of x^n from first principle

Sol Let $f(x) = x^n$

To find $\frac{dy}{dx}$ by using first principle

We know that

$$\frac{dy}{dx} = \underset{h\to 0}{Lt} \frac{f(x+h) - f(x)}{h} \longrightarrow ①$$

$$f(x+h) = (x+h)^n$$

From eq ① becomes

$$\left[\underset{x\to a}{Lt} \frac{x^n - a^n}{x-a} = na^{n-1} \qquad a = x,\ x = x+h\right]$$

$$\frac{dy}{dx} = \underset{h\to 0}{Lt} \frac{(x+h)^n - x^n}{h}$$

$$= \underset{x+h\to x}{Lt} \frac{(x+h)^n - x^n}{(x+h) - x}$$

$$\frac{dy}{dx} = nx^{n-1}$$

Formulas

1. $$\underset{x\to 0}{Lt} \frac{e^x - 1}{x} = 1$$

2. $$\underset{x\to 0}{Lt} \frac{a^x - 1}{x} = \log a$$

(B)

2. Find the derivative of e^x from first principle

Sol Let $f(x) = e^x$

To find $\frac{dy}{dx}$ by using first principle

We know that

$$\frac{dy}{dx} = \underset{h\to 0}{Lt} \frac{f(x+h) - f(x)}{h} \longrightarrow ①$$

$$f(x+h) = e^{x+h}$$

From eq ① becomes

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{e^{x+h} - e^{x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{x}\cdot e^{h} - e^{x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{x}(e^{h}-1)}{h} \qquad \left[\because \lim_{x\to 0} \frac{e^{x}-1}{x} = 1\right]$$

$$= e^{x} \lim_{h\to 0} \frac{e^{h}-1}{h}$$

$$= e^{x}(1)$$

$$\frac{dy}{dx} = e^{x} \;//$$

3. Find the derivative of a^{x} from the first principle

Sol let

$$f(x) = a^{x}$$

To find $\frac{dy}{dx}$ by using first principle

We know that

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \longrightarrow ①$$

$$f(x+h) = a^{x+h}$$

From eq ① becomes

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{a^{x+h} - a^{x}}{h}$$

$$= \lim_{h\to 0} \frac{a^{x}\cdot a^{h} - a^{x}}{h}$$

$$= \lim_{h\to 0} \frac{a^{x}(a^{h}-1)}{h} \qquad \left[\because \lim_{x\to 0} \frac{a^{x}-1}{x} = \log a\right]$$

$$= a^{x} \lim_{h\to 0} \frac{a^{h}-1}{h}$$

$$= a^{x}(\log a)$$

$$\frac{dy}{dx} = a^{x}\cdot \log a \;//$$

Find the maximum and minimum values of
$x^3-6x^2+12x-8$

Sol
Let $y = x^3-6x^2+12x-8 \longrightarrow ①$

Differentiation with respect to x in eq ①, we get

$$\frac{dy}{dx} = 3x^2-6(2x)+12(1)-0$$

$$\frac{dy}{dx} = 3x^2-12x+12 \longrightarrow ②$$

Assume $\frac{dy}{dx}=0$

$3x^2-12x+12=0$
$3(x^2-4x+4)=0$
$x^2-4x+4=0$
$x^2-2x-2x+4=0$
$x(x-2)-2(x-2)=0$
$(x-2)(x-2)=0$
$x-2=0 \quad x-2=0$
$x=2 \quad x=2$

Step-1

At $x=2$
Differentiation with respect to eq ②, we get

$$\frac{d^2y}{dx^2} = 3x^2-12x+12$$
$$= 3(2x)-12(1)+0$$
$$= 6x-12=0$$
$$= 6(2)-12=0$$
$$12-12=0$$

$\frac{d^2y}{dx^2}=0 \Rightarrow$ then the function is neither maximum or minimum.

~~step~~ $y = x^3-6x^2+12x-8$
$= (2)^3-6(2)^2+12(2)-8$
$= \cancel{8}-\cancel{24}+\cancel{24}-\cancel{8}$
$= 0$

$\underline{\sin^4 x}_{u}$ $\underline{\cos^3 x}_{v}$ differentiation

$$\frac{d}{dx}uv = u\frac{d}{dx}v + v\frac{d}{dx}u$$

$$= \sin^4 x \frac{d}{dx}\cos^3 x + \cos^3 x \cdot \frac{d}{dx}\sin^4 x$$

$$= \sin^4 x \cdot 3(\cos^2 x)(-\sin x) + \cos^3 x \, 4\sin^3 x \cos x$$

$$= -3\sin^5 x \cdot \cos^2 x + 4\cos^4 x \sin^3 x \; //$$

16/10/2019

UNIT - II

"Matrices and Determinant" - 14 M

Introduction

$A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ 6 & 11 \end{bmatrix}$

$3A + 5B + 2X = 0$ Find x

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow$ Row

$\downarrow$ Columns

2×2
$\downarrow$ Row $\downarrow$ colum

$a_{11} = a$, $a_{12} = b$; $a_{21} = c$, $a_{22} = d$

If $A = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 8 & 11 \\ 6 & 1 \end{bmatrix}$

then show that $(A+B)+C = A+(B+C)$

Sol Given

$A = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 8 & 11 \\ 6 & 1 \end{bmatrix}$

To show that $(A+B)+C = A+(B+C)$

L.H.S $= (A+B)+C$

$(A+B) = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 15 \\ 13 & 7 \end{bmatrix}$

$(A+B)+C = \begin{bmatrix} 7 & 15 \\ 13 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 11 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 26 \\ 19 & 8 \end{bmatrix}$ — ①

R.H.S $= A+(B+C)$

$(B+C) = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 11 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 19 \\ 13 & 3 \end{bmatrix}$

$A+(B+C) = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 11 & 19 \\ 13 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 26 \\ 19 & 8 \end{bmatrix}$ — ②

From eq ① & ②

L.H.S = R.H.S

$\therefore (A+B)+C = A+(B+C)$

If $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then find $4A - 5B$

Sol

Given

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$4A = 4\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 8 \\ 8 & 12 & 16 \\ 16 & 20 & 24 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 15 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$4A - 5B = \begin{bmatrix} 0 & 4 & 8 \\ 8 & 12 & 16 \\ 16 & 20 & 24 \end{bmatrix} - \begin{bmatrix} -5 & 10 & 15 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$4A - 5B = \begin{bmatrix} 5 & -6 & -7 \\ 8 & 7 & 16 \\ 16 & 20 & 29 \end{bmatrix} //$$

(b) If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ find x such that

$$2A + 3B - 2x = 0$$

Sol

Given

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 6 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 5 \\ 0 & -2 & 4 \end{bmatrix}$$

and $2A + 3B - 2X = 0 \longrightarrow (1)$

To find X

From eq (1)

$$2A + 3B = 2X$$

$$\frac{1}{2}(2A + 3B) = X \quad \text{— (2)}$$

$$2A = 2\begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 2 \\ 4 & 10 & 8 \\ 2 & 12 & 2 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 5 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 6 \\ -6 & 3 & 15 \\ 0 & -6 & 12 \end{bmatrix}$$

$$2A+3B = \begin{bmatrix} 2 & 6 & 2 \\ 4 & 10 & 8 \\ 2 & 12 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 6 \\ -6 & 3 & 15 \\ 0 & -6 & 12 \end{bmatrix}$$

$$2A+3B = \begin{bmatrix} 8 & 6 & 8 \\ -2 & 13 & 23 \\ 2 & 6 & 14 \end{bmatrix}$$

From eq ② be-comes.

$$X = \frac{1}{2}\begin{bmatrix} 8 & 6 & 8 \\ -2 & 13 & 23 \\ 2 & 6 & 14 \end{bmatrix} //$$

H.W If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find $(A+B)+C$ and $A+(B+C)$

HW If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 2 \\ 1 & 5 & 3 \\ 2 & 4 & 1 \end{bmatrix}$ then find $2A+5B$

H.W If $A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 2 & 1 \\ 1 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 1 & 1 & -2 \end{bmatrix}$ Find X such that

$2A+B+X=0 \qquad X = -2A-B$

Trace of Matrix :-

Ex:-1 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find trace of matrix 'A'.

Sol Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

To find trace of a matrix A

Trace of a matrix $A = 1+4 = 5$

If $A = \begin{bmatrix} 2 & 4 & 3 \\ 2 & -1 & 5 \\ 6 & 2 & 4 \end{bmatrix}$ then find trace of A.

Sol Given

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 2 & -1 & 5 \\ 6 & 2 & 4 \end{bmatrix}$$

To find trace of A

Trace of a matrix $A = 2+(-1)+4$

$= 5$ //

H.W If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$ such that $X = A+B$ then find the

$x_1 = 0$, $x_5 = -1$, $x_9 = 3$

Value of $x_1 + x_5 + x_9 = 2$

If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$ then find the

Values of $x-y+z-a$

Sol Given

$$\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$$

To find the values of $x-y+z-a$

$x-1=1$	$5-y=3$	$z-1=4$	$a-5=0$
$x=1+1$	$-y=3-5$	$z=4+1$	$a=5$
$x=2$	$\cancel{-}y=\cancel{-}2$	$z=5$	

$x-y+z-a = 2-2+5-5$

$= 0$ //

H.W If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -2 & a-4 \end{bmatrix}$ find the value of x, y, z, a.

Multiplication of two Matrices

If find the product of matrix

$$\begin{bmatrix} 3 & -2 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix}$$

Sol

$$\begin{bmatrix} 3 & -2 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix}$$ 1st row multiply with 1st column.

$$= \begin{bmatrix} 3\times4+(-2)\times2 & 3\times(-1)+(-2)\times5 \\ 1\times4+6\times2 & 1\times(-1)+6\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 12-4 & -3-10 \\ 4+12 & -1+30 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -13 \\ 16 & 29 \end{bmatrix}$$

H.W If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ find AB, BA

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ find AB, BA and verify commutative property $(AB = BA)$

Sol Given

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+0+3 & 0+(-2)+6 & 2+(-4)+0 \\ 2+0+(-1) & 0+3+(-2) & 4+6+0 \\ -3+0+2 & 0+1+4 & -6+2+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 4 & -2 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1+0+(-6) & (-2)+0+2 & 3+0+4 \\ 0+2+(-6) & 0+3+2 & 0+(-1)+4 \\ 1+4+0 & (-2)+6+0 & 3+(-2)+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -5 & 0 & 7 \\ -4 & 5 & 3 \\ 5 & 4 & 1 \end{bmatrix}$$

$\therefore AB \neq BA$

$$\begin{bmatrix} 4 & 4 & -2 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{bmatrix} \neq \begin{bmatrix} -5 & 0 & 7 \\ -4 & 5 & 3 \\ 5 & 4 & 1 \end{bmatrix}$$

$\therefore$ Commutative property doesn't exist.

H.W If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ then find A^3.

Sol Given

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

To find A^3

$$A^3 = A \cdot A^2 \longrightarrow ①$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+1\times5+3\times(-2) & 1\times1+1\times2+3\times(-1) & 1\times3+1\times6+3(-3) \\ 5\times1+2\times5+6\times(-2) & 5\times1+2\times2+6\times(-1) & 5\times3+2\times6+6(-3) \\ -2\times1+(-1)\times5+(-3)\times(-2) & -2\times1+(-1)\times2+(-3)\times(-1) & (-2)\times3+(-1)\times6+(-3)\times(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$A^3 = A \cdot A^2$$

$$= \begin{bmatrix} 1 & 1 & +3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times0+1\times3+3\times(-1) & 1\times0+1\times3+3\times(-1) & 1\times0+1\times9+3\times(-3) \\ 5\times0+2\times3+6\times(-1) & 5\times0+2\times3+6\times(-1) & 5\times0+2\times9+6\times(-3) \\ -2\times0+(-1)\times3+(-3)\times(-1) & -2\times0+(-1)\times3+(-3)\times(-1) & -2\times0+(-1)\times9+(-3)\times(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+3-3 & 0+3-3 & 0+9-9 \\ 0+6-6 & 0+6-6 & 0+18-18 \\ 0-3+3 & 0-3+3 & 0-9+9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

H.W Find the product of $A = \begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Sol Given

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

To find the product of AB

$$AB = \begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2+4 \\ 6+(-4)+3 \end{bmatrix} = \begin{bmatrix} 8 \\ +5 \end{bmatrix}$$

H.W

1. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 0 & 0 \end{bmatrix}$

find $(A+B)+C$ and $A+(B+C)$.

Sol Given

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find $(A+B)+C$ and $A+(B+C)$

$$(A+B) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 0 & 8 \\ 6 & 3 & 1 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 0 & 8 \\ 6 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 3 & 2 & 6 \\ 7 & -1 & 8 \\ 6 & 3 & 2 \end{bmatrix}$$

$$(B+C) = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 1 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 1 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 3 & 2 & 6 \\ 7 & -1 & 8 \\ 6 & 3 & 2 \end{bmatrix} //$$

H.W

2. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 2 \\ 1 & 5 & 3 \\ 2 & 4 & 1 \end{bmatrix}$ then find $2A+5B$

Sol Given

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 & 2 \\ 1 & 5 & 3 \\ 2 & 4 & 1 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 & -2 \\ 2 & 8 & 4 \\ 4 & 2 & 6 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} 4 & 5 & 2 \\ 1 & 5 & 3 \\ 2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 25 & 10 \\ 5 & 25 & 15 \\ 10 & 20 & 5 \end{bmatrix}$$

$$2A+5B = \begin{bmatrix} 4 & 6 & -2 \\ 2 & 8 & 4 \\ 4 & 2 & 6 \end{bmatrix} + \begin{bmatrix} 20 & 25 & 10 \\ 5 & 25 & 15 \\ 10 & 20 & 5 \end{bmatrix}$$

$$2A+5B = \begin{bmatrix} 24 & 31 & 8 \\ 7 & 33 & 19 \\ 14 & 22 & 11 \end{bmatrix} //$$

3. H.W If $A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 2 & 1 \\ 1 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ Find x such that $2A+B+X=0$

Sol Given

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 2 & 1 \\ 1 & 5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

To find x such that $2A+B+X=0$ — ①

To find X

From eq ①

$$2A+B+X=0$$

$$X = -2A-B \quad \text{— ②}$$

$$-2A = -2\begin{bmatrix} 2 & 1 & -3 \\ -1 & 2 & 1 \\ 1 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -2 & 6 \\ 2 & -4 & -2 \\ -2 & -10 & 0 \end{bmatrix}$$

$$-2A-B = \begin{bmatrix} -4 & -2 & 6 \\ 2 & -4 & -2 \\ -2 & -10 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$-2A-B = \begin{bmatrix} -5 & -5 & 8 \\ -2 & -4 & -3 \\ -3 & -11 & -2 \end{bmatrix}$$

From eq ② then X becomes

$$X = \begin{bmatrix} -5 & -5 & 8 \\ -2 & -4 & -3 \\ -3 & -11 & -2 \end{bmatrix} \quad //$$

4. H.W If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$ such that $X = A+B$ then find the value of $x_1 + x_5 + x_9$

Sol Given

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$$

such that $X = A+B$ then find the value of

$x_1 + x_5 + x_9$

$$X = A+B$$

$$X = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 3 \\ 5 & 1 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 3 \\ 5 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 3 \\ 5 & 1 & 3 \end{bmatrix}$$

$x_1 = 0$, $x_5 = -1$, $x_9 = 3$

$\therefore x_1 + x_5 + x_9 = 0 - 1 + 3 = 2$ //

H.W

5. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -2 & a-4 \end{bmatrix}$ find the value of x, y, z, a.

Sol

Given

$$\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -2 & a-4 \end{bmatrix}$$

To find the values of x, y, z, a.

$x-3=3$	$2y-8=2$	$z+2=-2$	$a-4=6$
$x=3+3$	$2y=2+8$	$z=-2-2$	$a=6+4$
$\boxed{x=6}$	$y=\frac{\cancel{10}\,5}{\cancel{2}}$	$\boxed{z=-4}$	$\boxed{a=10}$
	$\boxed{y=5}$		

H.W

6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ find AB, BA.

Sol

Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

To find AB and BA.

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3+2\times(-1) & 1\times2+2\times4 \\ 3\times3+4\times(-1) & 3\times2+4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 2+8 \\ 9-4 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 \\ 5 & 22 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1+2\times3 & 3\times2+2\times4 \\ -1\times1+4\times3 & -1\times2+4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+6 & 6+8 \\ -1+12 & -2+16 \end{bmatrix}$$

$$BA = \begin{bmatrix} 9 & 14 \\ 11 & 14 \end{bmatrix}$$

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If $A = \begin{bmatrix} 3 & 4 & 9 \\ 0 & -1 & 5 \\ 2 & 6 & 12 \end{bmatrix}$, $B = \begin{bmatrix} 13 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ Find AB

Sol Given

$$A = \begin{bmatrix} 3 & 4 & 9 \\ 0 & -1 & 5 \\ 2 & 6 & 12 \end{bmatrix}, \quad B = \begin{bmatrix} 13 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$\underset{R \;\; C}{3\times3} \qquad \underset{R \;\; C}{2\times3}$$

To find AB

$$\underset{\text{Column}}{A} = \underset{\text{Row}}{B}$$

[column of matrix of $A \neq$ Row of matrix of B].

$$\underset{\text{column}}{A} \neq \underset{\text{Row}}{B}$$

$\therefore$ The product of AB is not exists.

(B) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then prove that $A^2 - 4A - 5I = 0$

I = Identity matrix

$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol Given

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

To prove that $A^2 - 4A - 5I = 0$

$L.H.S = A^2 - 4A - 5I \longrightarrow ①$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$4A = 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$5I = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

from eq ① becomes

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow A^2 - 4A - 5I = 0$$

L.H.S = R.H.S

If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$ find k.

Sol

Given

$$A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$$

and $A^2 = 0$ — ①

To find k.

$A^2 = A \cdot A$

$$A^2 = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4-4 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix}$$

From eq ① becomes

$$\begin{bmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$-2-k = 0$

$-2 = k \qquad \therefore \boxed{k = -2}$

Transpose of a Matrix A^T (or) A' (or) $\bar{A}$

1. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & 0 & 3 \end{bmatrix}$ find A^T

Sol Given

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & 0 & 3 \end{bmatrix}$$

To find A^T [Row is converted to column]

$$A^T = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 7 & 0 \\ 4 & 8 & 3 \end{bmatrix}$$

(B)

2. If $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} +1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$

Symmetric Matrix :- $A^T = A \rightarrow$ Ex:- $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} = \begin{bmatrix} a \\ h \\ g \end{bmatrix}$

Skew - Symmetric Matrix :- $A^T = -A$

Ex:- $\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

H.W If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then show that $(aI+bE)$

$a^3 I + 3a^2 bE$

Sol Given

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show that $(aI+bE)^3 = a^3 I + 3a^2 bE$

L.H.S $(aI+bE)^3$

$aI = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

$bE = b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$(aI+bE) = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$(aI+bE)^3 = (aI+bE)^2 (aI+bE) \longrightarrow (1)$

$(aI+bE)^2 = (aI+bE)(aI+bE)$

$= \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$= \begin{bmatrix} a^2+0 & ab+ba \\ 0+0 & 0+a^2 \end{bmatrix}$

$(aI+bE)^2 = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$

From eq (1) becomes

$(aI+bE)^3 = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$= \begin{bmatrix} a^3+0 & a^2b+2a^2b \\ 0+0 & 0+a^3 \end{bmatrix}$

$$(aI+bE)^3 = \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix} \quad -②$$

R.H.S $\quad a^3 I + 3a^2 bE$

$$a^3 I = a^3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^3 & 0 \\ 0 & a^3 \end{bmatrix}$$

$$3a^2bE = 3a^2b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3a^2b \\ 0 & 0 \end{bmatrix}$$

$$a^3 I + 3a^2bE = \begin{bmatrix} a^3 & 0 \\ 0 & a^3 \end{bmatrix} + \begin{bmatrix} 0 & 3a^2b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix} \quad -③$$

from eq ② & ③ $\quad$ L.H.S = R.H.S

$$\therefore (aI+bE)^3 = a^3 I + 3a^2bE$$

H.W If $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} +1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$

Sol

Given

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$$

To verify that $(AB)^T = B^T \cdot A^T$

$$AB = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+(-1)\times-3+2\times5 & 2\times-2+0+2\times4 \\ 1\times1+3\times(-3)+(-4)\times5 & 1\times-2+0+(-4)\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3+10 & -4+0+8 \\ 1-9-20 & -2+0-16 \end{bmatrix}$$

$$AB = \begin{bmatrix} 15 & 4 \\ -28 & -18 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 15 & -28 \\ 4 & -18 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 0 & 4 \end{bmatrix}.$$

$$A^T = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

$$B^T.A^T = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 0 & 4 \end{bmatrix} . \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3+10 & 1-9-20 \\ -4+0+8 & -2+0-16 \end{bmatrix}$$

$$B^T.A^T = \begin{bmatrix} 15 & -28 \\ 4 & -18 \end{bmatrix}$$

$$\therefore (AB)^T = B^T.A^T$$

$$\therefore \begin{bmatrix} 15 & -28 \\ 4 & -18 \end{bmatrix} = \begin{bmatrix} 15 & -28 \\ 4 & -18 \end{bmatrix} //$$

Adjoint of a matrix:-

$$\text{Adj } A = [\text{co-factor matrix}]^T$$

$$\det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-b$$

(B) Find Adjoint of a matrix $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

Sol Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

To find Adj A

We know that

$$\text{Adj } A = [\text{co-factor matrix}]^T$$

$$\text{cofactor } 2 = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = (0)(1) - (1)(2)$$
$$0 - 2 = -2$$

$$\text{cofactor } 1 = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$$

$$\text{cofactor } 2 = \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2-0 = 2$$

$$\text{cofactor } 1 = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1-4 = -3$$

$$\text{cofactor } 0 = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} = 2-4 = -2$$

cofactor 1 $= \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = 4-2 = 2$

cofactor 2 $= \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$

cofactor 2 $= \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 2-2 = 0$

cofactor 1 $= \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$

$$= \begin{vmatrix} -2 & -1 & 2 \\ -3 & -2 & 2 \\ 1 & 0 & -1 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

cofactor matrix

$$= \begin{vmatrix} -2 & 1 & 2 \\ 3 & -2 & -2 \\ 1 & 0 & -1 \end{vmatrix}$$

Adj A = [co-factor matrix]^T

$$= \begin{vmatrix} -2 & 3 & 1 \\ 1 & -2 & 0 \\ 2 & -2 & -1 \end{vmatrix} \quad //$$

H.W

Find Adjoint of a matrix $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Sol

Let $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ $\quad$ det $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

To find Adj A

We know that Adj A = [co-factor matrix]^T

cofactor $-1 = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = 1-4 = -3$

cofactor $-2 = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = 2-(-4) = 2+4 = 6$

cofactor $-2 = \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix} = -4-2 = -6$

Cofactor $2 = \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix} = -2-4 = -6$

Cofactor $1 = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1-(-4) = -1+4 = 3$

Cofactor $-2 = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 2+(-4) = 2+4 = 6$

Cofactor $2 = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 4-(-2) = 4+2 = 6$

Cofactor $-2 = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 2-(-4) = 2+4 = 6$

Cofactor $1 = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1-(-4) = -1+4 = 3$

$$= \begin{vmatrix} -3 & 6 & -6 \\ -6 & 3 & 6 \\ 6 & 6 & 3 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Cofactor matrix

$$= \begin{vmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{vmatrix}$$

Adj A = [co-factor matrix]^T

$$= \begin{vmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{vmatrix}$$

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Inverse of a Matrix :-

$$A^{-1} = \frac{1}{\det A} \cdot \text{Adj } A$$

1. Find the inverse of a matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Sol Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

To find A^{-1}

We know that

$$A^{-1} = \frac{1}{\det A} \cdot \text{Adj } A \longrightarrow (1)$$

$$\det A = 1[16-9] - 3[4-3] + 3[3-4] \qquad \left[\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad-bc \right]$$

$$= 7-3-3 = 1$$

$$\boxed{\det A = 1}$$

We know that

$$\text{Adj. } A = [\text{co-factor matrix}]^T$$

From eq (1)

$$A^{-1} = \frac{1}{1}(\quad)$$

$$\text{cofactor } 1 = \begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} = 16-9 = 7$$

$$\text{cofactor } 3 = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 4-3 = 1$$

$$\text{cofactor } 3 = \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} = 3-4 = -1$$

$$\text{cofactor } 1 = \begin{vmatrix} 3 & 3 \\ 3 & 4 \end{vmatrix} = 12-9 = 3$$

$$\text{cofactor } 4 = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 4-3 = 1$$

$$\text{cofactor } 3 = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 3-3 = 0$$

$$\text{cofactor } 1 = \begin{vmatrix} 3 & 3 \\ 4 & 3 \end{vmatrix} = 9-12 = -3$$

$$\text{cofactor } 3 = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 3-3 = 0$$

$$\text{cofactor } 4 = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 4-3 = 1$$

$$= \begin{vmatrix} 7 & 1 & -1 \\ 3 & 1 & 0 \\ -3 & 0 & 1 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

cofactor matrix

$$= \begin{vmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{vmatrix}$$

Adj A = (co-factor matrix)$'$

$$= \begin{vmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

From eq (1) becomes

$$A^{-1} = \frac{1}{1} \cdot \begin{vmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}.$$

<u>Formula</u>

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then Adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Find adjoint of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Adjoint } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Crammer's rule :-

General form of equation

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

By using Crammer's Rules

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

Solve the following simultaneous equations by using 10 M

(8) Crammer's method

$3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$

Sol Given

$$3x + 4y + 5z = 18$$
$$2x - y + 8z = 13$$
$$5x - 2y + 7z = 20$$

To find x, y, z by using Crammer's method we have

$$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta},\ z = \frac{\Delta_3}{\Delta} \longrightarrow ①$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{vmatrix} = 3(-7+16) - 4(14-40) + 5(-4+5)$$
$$= 3(9) - 4(-26) + 5(1)$$
$$= 27 + 104 + 5 = 136$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta_1 = \begin{vmatrix} 18 & 4 & 5 \\ 13 & -1 & 8 \\ 20 & -2 & 7 \end{vmatrix} = 18(-7-(-16)) - 4(91-160) + 5(-26+20)$$
$$= 18(-7+16) - 4(-69) + 5(-6)$$
$$= 162 + 276 - 30$$
$$= 438 - 30$$
$$= 408$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta_2 = \begin{vmatrix} 3 & 18 & 5 \\ 2 & 13 & 8 \\ 5 & 20 & 7 \end{vmatrix} = 3(91-160) - 18(14-40) + 5(40-65)$$
$$= 3(-69) - 18(-26) + 5(-25)$$
$$= -207 + 468 - 125$$
$$= 468 - 332$$
$$= 136$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta_3 = \begin{vmatrix} 3 & 4 & 18 \\ 2 & -1 & +13 \\ 5 & -2 & 20 \end{vmatrix} = 3(-20+26) - 4(40-65) + 18(-4+5)$$
$$= 3(6) - 4(-25) + 18(1)$$
$$= 18 + 100 + 18$$
$$= 136$$

$= 6(-2)+3+1(7)$

$= -12+3+7 = -12+10 = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 9 & 3 \end{vmatrix} = 1(6-9)-6(3-2)+1(9-4)$$

$= 1(-3)-6(1)+1(5)$

$= -3-6+5 = -9+5 = -4$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & -1 & 9 \end{vmatrix} = 1(-9-(-2))-1(9-4)+6(-1-(-2))$$

$= 1(-9+2)-1(5)+6(-1+2)$

$= 1(-7) -5+6(1)$

$= -7-5+6 = -12+6 = -6$

Substitute $\Delta, \Delta_1, \Delta_2, \Delta_3$ values eq ①, we have

$x = \frac{\Delta_1}{\Delta} = \frac{\cancel{-2}}{\cancel{-2}} = 1$, $y = \frac{\Delta_2}{\Delta} = \frac{\cancel{-4}\,2}{\cancel{-2}} = 2$, $z = \frac{\Delta_3}{\Delta} = \frac{\cancel{-6}}{\cancel{-2}} = 3$

$\therefore x=1, y=2, z=3$

Check

$x+y+z = 6$

$1+2+3 = 6$

$3+3 = 6$

$6 = 6$ ||

29/10/2019

2M

① If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{bmatrix}$ find minor of 6

Sol Given

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{bmatrix}$$

To find minor of 6

Minor of 6 $= \begin{vmatrix} 1 & 2 \\ 2 & -7 \end{vmatrix}$ det A = ad − bc.

$= -7-4 = -11$

① If $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ find cofactor of 0

Sol Given

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

To find cofactor of $0 = +\begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$ $\det A = ad - bc$

$$= 2 - 4 = -2 \;//$$

Singular matrix:

$$\det A = 0$$

Non-singular matrix:

$$\det A \neq 0$$

Matrix inverse method (or) Matrix method

General equation

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

By using matrix inverse method, we have

$$X = A^{-1}B$$

where

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

30/10/2019

① Solve the following equation by using matrix inverse method. $x+y+z=6$, $x-y+z=2$, $2x-y+3z=9$

Sol Given

$$x+y+z=6$$
$$x-y+z=2$$
$$2x-y+3z=9$$

To find x, y, z by matrix inverse method, we have

$$X = A^{-1} B \rightarrow (1)$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad A = \begin{bmatrix} \overset{+}{1} & \overset{-}{1} & \overset{+}{1} \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$$

First to find A^{-1}

We know that

$$A^{-1} = \frac{1}{\det A} \cdot \operatorname{Adj} A \rightarrow (2)$$

$$\det A = 1[-1\times 3 - 1\times(-1)] - 1[1\times 3 - 1\times 2] + 1[1\times(-1) - 2\times(-1)]$$

$$= 1[-3+1] - 1[3-2] + 1[-1+2]$$

$$= 1[-2] - 1[1] + 1[1]$$

$$\det A = -2-1+1 = -3+1 = -2 \;//$$

We know that

$$\operatorname{Adj} A = [\text{co-factor matrix}]^T \qquad [ad-bc]$$

cofactor $1 = \begin{bmatrix} -1 & 1 \\ -1 & 3 \end{bmatrix} = -3+1 = -2$

cofactor $1 = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = 3-2 = 1$

cofactor $1 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = -1+2 = 1$

cofactor $1 = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} = 3+1 = 4$

cofactor $-1 = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = 3-2 = 1$

cofactor $1 = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} = -1-2 = -3$

cofactor $2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = 1+1 = 2$

cofactor $-1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 1-1 = 0$

cofactor $3 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = -1-1 = -2$

~~cofactor matrix~~

$$\begin{bmatrix} -2 & 1 & 1 \\ 4 & 1 & -3 \\ 2 & 0 & -2 \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

cofactor matrix $\begin{vmatrix} -2 & -1 & 1 \\ -4 & 1 & +3 \\ 2 & 0 & -2 \end{vmatrix}$

Adj.A = [co-factor matrix]

$$= \begin{bmatrix} -2 & -4 & 2 \\ -1 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$$

From eq ② becomes

$$A^{-1} = \frac{+1}{-2} \begin{vmatrix} -2 & -4 & 2 \\ -1 & 1 & 0 \\ 1 & 3 & -2 \end{vmatrix}$$

$$A^{-1} = \frac{-1}{+2} \begin{vmatrix} -2 & -4 & 2 \\ -1 & 1 & 0 \\ 1 & 3 & -2 \end{vmatrix}$$

$$X = A^{-1}B = -\frac{1}{2} \begin{bmatrix} -2 & -4 & 2 \\ -1 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -12-8+18 \\ -6+2+0 \\ 6+6-18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x=1,\ y=2,\ z=3$

$x+y+z=6$

$1+2+3=6$

$6=6$

Characteristic Equation :-

$$|A-\lambda I|=0$$

(3) 1. $A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$ then find characteristic equation and characteristic root (or) eign values.

Sol Given

$$A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$$

To find characteristic equation we know that

characteristic equation $|A-\lambda I|=0$

$$A-\lambda I=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}$$

$$A-\lambda I=\begin{bmatrix}\overset{+}{1-\lambda}&\overset{-}{0}&\overset{+}{-1}\\1&2-\lambda&1\\2&2&3-\lambda\end{bmatrix}$$

$[ad-bc]$

To find $|A-\lambda I|=0$

$$=(1-\lambda)[(2-\lambda)(3-\lambda)-2]-0[-1[2-2(2-\lambda)]=0$$

$$=(1-\lambda)[6-2\lambda-3\lambda+\lambda^2-2]-[2-4+2\lambda]=0$$

$$=(1-\lambda)[4-5\lambda+\lambda^2]-(-2+2\lambda)=0$$

$$=4-5\lambda+\lambda^2-4\lambda+5\lambda^2-\lambda^3+2-2\lambda=0$$

$$=-\lambda^3+6\lambda^2-11\lambda+6=0$$

$$=-(\lambda^3-6\lambda^2+11\lambda-6)=0$$

$$=\lambda^3-6\lambda^2+11\lambda-6=0$$

It is a characteristic equation.

To find characteristic roots (or) eign values.

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$$\begin{array}{c|cccc} \lambda=1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$1\lambda^2 - 5\lambda + 6 = 0$

$1 \times 6 = 6$ Factors

$-3 \; -2$

i. two no. multiply = 6

ii. two no. add = -5

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$

$\lambda(\lambda-3) - 2(\lambda-3) = 0$

$(\lambda-3)(\lambda-2) = 0$

$\lambda - 3 = 0$ | $\lambda - 2 = 0$

$\lambda = 3$ | $\lambda = 2$

$\therefore$ Characteristic root is

$\lambda = 1, 2, 3$

2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find characteristic equation and eign values.

3. If $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ then find characteristic equation and eign values.

Cayley - Hamilton theorem :-

(B) Every square matrix satisfies its characteristic equations

1. Verify the Cayley - Hamilton theorem for the square matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and hence find its inverse also.

Sol Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Characteristic equation $|A - \lambda I| = 0$

$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$

$|A - \lambda I| = 0$ [ad - bc]

$\Rightarrow (1-\lambda)(4-\lambda) - 6 = 0$

$\Rightarrow 4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$

$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$

To verify cayley - Hamilton theorem.

$$A^2 - 5A - 2I = 0 \quad \text{--- (1)}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^2 - 5A - 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 5A - 2I = 0$$

L.H.S = R.H.S

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To find A^{-1}

$$A^2 - 5A - 2I = 0$$

multiply A^{-1} on both sides, we get

$$A^{-1}[A^2 - 5A - 2I] = 0$$

$$A^{-1} \cdot A^2 - 5A^{-1}A - 2IA^{-1} = 0$$

$$A - 5I - 2A^{-1} = 0$$

$$2A^{-1} = A - 5I \qquad \begin{bmatrix} AA^{-1} = A^{-1}A = I \\ 2A^{-1} = A^{-1}2 = A^{-1} \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}[A - 5I] \rightarrow (2)$$

$$A - 5I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A - 5I = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

From eq ② becomes

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$ //

(B)

1. $A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$ Find A^8 using cayley Hamilton theorem

Sol Given

$$A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

To find A^8 by using caylay Hamilton theorem, we have

Now, characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ -2 & -1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(-1-\lambda)+2 = 0$$

$$\cancel{-2} - 2\lambda + \lambda + \lambda^2 \cancel{+2} = 0$$

$$\lambda^2 - \lambda = 0$$

By using caylag Hamilton theorem, we have

$$A^2 - A = 0$$

$$A^2 = A \longrightarrow ①$$

To find A^8

$$A^8 = (A^4)^2$$

$$= (A^2 \cdot A^2)^2$$

$$= (A \cdot A)^2 \quad [\text{eq } ①]$$

$$= (A^2)^2 \quad [\text{eq } ①]$$

$$= (A)^2$$

$$= A$$

$$A^8 = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

(5) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then show that $A^5 - 5A^4 - 10A^2 - 4A = 0$ by using cayley-Hamilton theorem

Sol Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

characteristic equation $|A - \lambda I| = 0$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow (1-\lambda)(4-\lambda) - 6 = 0$$

$$\Rightarrow 4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$$

$$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

By using cayley-Hamilton theorem.

$$A^2 - 5A - 2I = 0$$

$$A^2 - 5A = 2I \quad \text{——} (1)$$

To show that $A^5 - 5A^4 - 10A^2 - 4A = 0$

$$L.H.S = A^5 - 5A^4 - 10A^2 - 4A$$

$$= A^3[A^2 - 5A] - 10A^2 - 4A$$

$$= A^3[2I] - 10A^2 - 4A \quad [\because eq\ (1)] \quad [A^3 I = A^3]$$

$$= 2A^3 - 10A^2 - 4A$$

$$= 2A(A^2 - 5A) - 4A$$

$$= 2A(2I) - 4A$$

$$= 4A - 4A$$

$$= 0 = R.H.S$$

$$\therefore A^5 - 5A^4 - 10A^2 - 4A = 0 \ //$$

Determinants

1. Find the determinant (i) $\begin{vmatrix} i & 0 \\ 0 & -i \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Sol (i). $\begin{vmatrix} i & 0 \\ 0 & -i \end{vmatrix}$ $[i^2 = -1]$

$= -i^2 - 0$

$= -(-1)$

$= 1$

(ii.) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$ $\begin{bmatrix} \omega^3 = 1 \\ 1+\omega+\omega^2 = 0 \end{bmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

$= 0$ // [any row (or) any column contains zero then det is zero]

(8) Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol L.H.S $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$

$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$ $a^2 - b^2 = (a-b)(a+b)$

$= \begin{vmatrix} 1 & a & a^2 \\ 0 & (b-a) & (b-a)(b+a) \\ 0 & (c-a) & (c-a)(c+a) \end{vmatrix}$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= (-)(a-b)(c-a)\left[1(c+a-(b+a))-0+0\right]$$

$$= (-)(a-b)(c-a)\left[c+\cancel{a}-b-\cancel{a}\right]$$

$$= (-)(a-b)(c-a)(-)(b-c)$$

$$= (a-b)(b-c)(c-a) = R\cdot H\cdot S$$

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a) \quad //$$

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(B) Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

$R_1+R_2+R_3$

$a+b+4c,\ b+c+4a$

Sol

$$L\cdot H\cdot S = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$C_1+C_2+C_3$

$2a+2b+2c$

$2a+2b+2c$

$$C_1 \longrightarrow C_1+C_2+C_3$$

$$= \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$

$$2a+2b+2c \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$R_2 \to R_2-R_1,\ R_3 \to R_3-R_1$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

$$= 2(a+b+c)\left[1(a+b+c)^2-0\right]$$

$= 2(a+b+c)^3 = R.H.S$

$$\therefore \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

H.W (B) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

HW (B) Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

(B) If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and a, b, c are different and non-zero then show that $abc = -1$.

Sol Given

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$C_3 \longleftrightarrow C_1$

$$= \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$C_3 \longleftrightarrow C_2$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (1+abc) = 0$$

Since $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$

$\Rightarrow 1+abc = 0$

$\Rightarrow abc = -1$ //

Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Sol: L.H.S $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

$$= \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} + \begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$C_3 \leftrightarrow C_1$

$$= \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} + \begin{vmatrix} b & a & c \\ c & b & a \\ a & c & b \end{vmatrix}$$

$C_3 \leftrightarrow C_2$

$$= \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= 2 \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$C_3 \leftrightarrow C_1$

$$= 2 \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = R.H.S$$

$$\therefore \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

i. Show that $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$ $\quad R_1 \to R_1 - R_2 - R_3$

ii. Find the determinants $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

iii. Find show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & b & a \end{vmatrix}^2 = (a^3+b^3+c^3-3abc)^2$ $\quad R_1 \to R_1+R_2+R_3$

iv. Find the values of x if $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ $\quad R_1 \to R_1+R_2+R_3$ $\quad x = 2/3, 11/3$

H.W

i. Show that $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$

Sol Given

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$$

L.H.S

$R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} 0 & -2z & -2y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$$= 0\left[(z+x)(x+y)-yz\right] + 2z\left[(x+y)y - yz\right] - 2y\left[yz - z(z+x)\right]$$

$$= 0 + 2z[xy + y^2 - yz] - 2y[yz - z^2 - xz]$$

$$= 2xyz + \cancel{2y^2z} - \cancel{2yz^2} - \cancel{2y^2z} + \cancel{2yz^2} + 2xyz = 4xyz$$

$$\therefore \begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz \;//$$

H.W

ii. Find the determinants $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

Sol Given

$$\begin{vmatrix} \overset{+}{a} & \overset{-}{h} & \overset{+}{g} \\ h & b & f \\ g & f & c \end{vmatrix}$$

$$= a[(bc - f^2)] - h[hc - gf] + g[hf - gb]$$

$$= abc - af^2 - h^2c + hgf + gfh - g^2b$$

$$= abc + 2gfh - af^2 - bg^2 - ch^2 \;//$$

H.W

iii. Find show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = (a^3 + b^3 + c^3 - 3abc)^2$

Sol Given

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = (a^3 + b^3 + c^3 - 3abc)^2$$

L.H.S

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$$

$$= abc - a^3 - b^3 + abc + abc - c^3$$

$$= -a^3 - b^3 - c^3 + 3abc$$

$$= (a^3 + b^3 + c^3 - 3abc)^2 \;//$$

H.W Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Sol

Given

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$L.H.S = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c. & c-a-b \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \Rightarrow C_2 - C_1$

$C_3 \Rightarrow C_3 - C_1$

$$\Rightarrow (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$(a+b+c)\ (a+b+c)\ (a+b+c) = (a+b+c)^3$$

$$\therefore \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$\therefore$ L.H.S = R.H.S

H.W show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol Given $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

$R_2 \to R_2 - R_1$ L.H.S

$R_3 \to R_3 - R_1$ $\begin{vmatrix} bc & b+c & 1 \\ ca-bc & a-b & 0 \\ ab-bc & a-c & 0 \end{vmatrix} = \begin{vmatrix} bc & b+c & 1 \\ c(a-b) & a-b & 0 \\ b(a-c) & a-c & 0 \end{vmatrix}$

$$\Rightarrow (a-b)(a-c)\begin{vmatrix} bc & b+c & 1 \\ c & 1 & 0 \\ b & 1 & 0 \end{vmatrix} \quad R_3 \to R_3 - R_2$$

$$\Rightarrow (a-b)(a-c)\begin{vmatrix} bc & b+c & 1 \\ c & 1 & 0 \\ b-c & 0 & 0 \end{vmatrix}$$

$$\Rightarrow (a-b)(a-c)(1(0-(b-c)) \Rightarrow (a-b)(a-c)(b-c)$$

$\therefore$ L.H.S = R.H.S

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a),$$

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find characteristic equation and eign values.

Sol Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

To find characteristic equation we know that characteristic equation $|A - \lambda I| = 0$

$$A - \lambda I = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix}$$

$$A - \lambda I = \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix}$$

To find $|A - \lambda I| = 0$

$$= (1-\lambda)(4-\lambda) - 2(3) = 0$$

$$= (1-\lambda)(4-\lambda) - 6 = 0$$

$4-\lambda-4\lambda+\lambda^2-6=0$

$4-5\lambda+\lambda^2-6=0$

$+\lambda^2-5\lambda-2=0$

$+(\lambda^2-5\lambda-2)=0$

$\lambda^2-5\lambda-2=0$ //

$\lambda = \frac{5\pm\sqrt{25+8}}{2} = \frac{5\pm\sqrt{33}}{2}$

$\therefore$ eign values are $\frac{5+\sqrt{33}}{2}$, $\frac{5-\sqrt{33}}{2}$

(B) iv. Find the value of x if $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Sol Given

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

L.H.S

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$$

$\therefore R_1+R_2+R_3$

$= 3x-8+6$

$= 3x-2$

$$\begin{vmatrix} 3x-2 & 3x-2 & 3x-2 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$$

$$3x-2 \begin{vmatrix} 1 & 1 & 1 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$3x-8-3$

$C_3 \to C_3 - C_1$

$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$

$$(3x-2) \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3x-11 & 0 \\ 3 & 0 & 3x-11 \end{vmatrix}$$

$= (3x-2)\left[1\left((3x-11)(3x-11)-0\right]\right.$

$= 3x-2\left[1\left(9x^2-33x-33x\right.\right.$

If $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ then find characteristic equation and eign values.

Sol Given $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

To find characteristic equation

We know that

characteristic equation $|A-\lambda I| = 0$

Take $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 2 & 3 \\ 1 & -\lambda & -1 \\ 1 & 2 & 1-\lambda \end{bmatrix}$$

Characteristic equation is $|A-\lambda I| = 0$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{bmatrix} 2-\lambda & 2 & 3 \\ 1 & -\lambda & -1 \\ 1 & 2 & 1-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(-\lambda+\lambda^2\cancel{+2}) - 2(1-\lambda+1) + 3(2+\lambda) = 0$$

$$\Rightarrow -2\lambda + 2\lambda^2\cancel{+4} + \lambda^2 - \lambda^3\cancel{-2\lambda-4} + 2\lambda + 6 + 3\lambda = 0$$

$$\Rightarrow -\lambda^3 + 3\lambda^2 + \lambda + 6 = 0 \Rightarrow -(\lambda^3 - 3\lambda^2 - \lambda - 6) = 0$$

$$= \lambda^3 - 3\lambda^2 - \lambda - 6 = 0$$

$$= \lambda^3 - 3\lambda^2 - \lambda - 6 = 0$$

28/09/2019

OM

Basic Formulas

$\Rightarrow \; - \times - \; = +$

$\Rightarrow \; + \times - \; = -$

$\Rightarrow \; - \times + \; = -$

$\Rightarrow \; + \times + \; = +$

$\Rightarrow \; (a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow \; (a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow \; (a^2 - b^2) = (a+b)\,(a-b)$

(or)

$(a-b)\,(a+b)$

$\Rightarrow \; (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\Rightarrow \; (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\Rightarrow \; (a^3 + b^3) = (a+b)\,(a^2 \overset{(-)}{\cancel{+}} ab + b^2)$

$\Rightarrow \; (a^3 - b^3) = (a-b)\,(a^2 + ab + b^2)$

1/10/2019

UNIT - I

Partial Fraction [10 M]

Method - I

B Resolve $\frac{3x}{(x-2)(x+1)}$

Sol

$$\frac{3x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \longrightarrow ①$$

To find A, B Values

$$\frac{3x}{\cancel{(x-2)(x+1)}} = \frac{A(x+1) + B(x-2)}{\cancel{(x-2)(x+1)}}$$

$$3x = A(x+1) + B(x-2) \longrightarrow ②$$

To find A

Put $x = 2$, we get

$$3(2) = A(2+1) + B(2-2)$$

$$6 = 3A + 0$$

$$\frac{\cancel{6}^{2}}{\cancel{3}} = A$$

$$\therefore \boxed{A = 2}$$

From equation ②, To find B

Put $x = -1$, we get

$$3(-1) = A(-1+1) + B(-1-2)$$

$$-3 = 0 - 3B$$

$$\cancel{-}3 = \cancel{-}3B$$

$$\frac{\cancel{3}}{\cancel{3}} = B$$

$$\therefore \boxed{B = 1}$$

Substitute A, B values in eq (1) we get

$$\frac{3x}{(x-2)(x+1)} = \frac{2}{x-2} + \frac{1}{x+1}$$

H.W Resolve $\frac{1}{(x-4)(x-9)}$

$A = -\frac{1}{5}$, $B = 1/5$

Sol

$$\frac{1}{(x-4)(x-9)} = \frac{A}{(x-4)} + \frac{B}{(x-9)} \quad \text{—— (1)}$$

To find A, B values

$$\frac{1}{\cancel{(x-4)(x-9)}} = \frac{A(x-9)+B(x-4)}{\cancel{(x-4)(x-9)}}$$

$$1 = A(x-9)+B(x-4) \quad \text{—— (2)}$$

To find A

Put $x=4$, we get

$1 = A(x-9)+B(x-4)$

$1 = A(4-9)+B(4-4)$

$1 = A(-5)+0$

$1 = -5A$

$\boxed{A = -\frac{1}{5}}$

From equation (2), To find 'B'

Put $x=9$, we get

$1 = A(x-9)+B(x-4)$

$1 = A(9-9)+B(9-4)$

$1 = 0 + B(5)$

$1 = 5B$

$1/5 = B$

$\boxed{B = 1/5}$

Substitute A, B values in eq ① we get

$$\frac{1}{(x-4)(x-9)} = \frac{-1/5}{(x-4)} + \frac{1/5}{(x-9)}$$

$$= \frac{-1}{5(x-4)} + \frac{1}{5(x-9)}$$

Method - II

(8) Resolve $\frac{2x+3}{3x^2+14x-5}$ into partial fractions

$$\frac{2x+3}{3x^2+14x-5} = \frac{2x+3}{3x^2+15x-1x-5}$$

$$= \frac{2x+3}{3x(x+5)-1(x+5)}$$

$$= \frac{2x+3}{(x+5)(3x-1)}$$

Hint

$3x-5 = -15$

15 -1

(i) two number multiply

(ii) two number add (+14)

$$\frac{2x+3}{3x^2+14x-5} = \frac{2x+3}{(x+5)(3x-1)} \quad \text{—— ①}$$

Consider

$$\frac{2x+3}{(x+5)(3x-1)} = \frac{A}{(x+5)} + \frac{B}{(3x-1)} \quad \text{—— ②}$$

$$\frac{2x+3}{\cancel{(x+5)(3x-1)}} = \frac{A(3x-1)+B(x+5)}{\cancel{(x+5)(3x-1)}}$$

$$2x+3 = A(3x-1)+B(x+5) \quad \text{—— ③}$$

To find A, B values

Find 'A'

Put $x=-5$, we get

$$2(-5)+3 = A[3(-5)-1]+0$$

$$-10+3 = A(-15-1)$$

$$\cancel{-}7 = \cancel{-}16A$$

$$7/16 = A$$

$$\boxed{A = \frac{7}{16}}$$

From equation ③, To find B

Put $x = \frac{1}{3}$, we get

$$2(1/3)+3 = 0 + B(1/3+5)$$

$$\frac{2+9}{3} = B\left(\frac{1+15}{3}\right)$$

$$\frac{11}{\cancel{3}} = \frac{16}{\cancel{3}}B$$

$$\frac{11}{16} = B$$

$$\therefore \boxed{B = \frac{11}{16}}$$

Hint

$3x-1=0$

$3x=1$

$x=\frac{1}{3}$

Substitute A, B values in eq ②, we get

$$\frac{2x+3}{(x+5)(3x-1)} = \frac{7/16}{x+5} + \frac{11/16}{(3x-1)}$$

$$= \frac{7}{16(x+5)} + \frac{11}{16(3x-1)}$$

Resolve (i) $\frac{x-4}{x^2-5x+6}$ (ii) $\frac{2x-1}{(x-1)(2x+3)}$

Method-II

1. Resolve $\frac{1}{x^2(x+2)}$ into partial fractions.

Sol

$$\frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} \longrightarrow ①$$

$$\frac{1}{\cancel{x^2(x+2)}} = \frac{Ax(x+2)+B(x+2)+Cx^2}{\cancel{x^2(x+2)}}$$

$$1 = Ax(x+2)+B(x+2)+Cx^2 \longrightarrow ②$$

$\frac{x^{\cancel{2}}(x+2)}{\cancel{x}} = x(x$

$\frac{\cancel{x^2}(x+2)}{\cancel{x^2}} = x+2$

$\frac{x^2\cancel{(x+2)}}{\cancel{x+2}} = x^2$

To find A, B, C values

Put $x = 0$ in eq ②, we get

$$1 = 0 + B(0+2) + 0$$

$$1 = 2B$$

$$1/2 = B$$

$$\therefore \boxed{B = 1/2}$$

From eq ② put $x = -2$, we get

$$1 = A(-2)(-2+2) + B(-2+2) + C(-2)^2$$

$$1 = 0 + 0 + 4C$$

$$1 = 4C$$

$$1/4 = C$$

$$\therefore \boxed{C = \frac{1}{4}}$$

From eq ② simplify then find the value of A, we get

$$1 = Ax^2 + 2Ax + Bx + 2B + Cx^2$$

Compare 'x^2' coefficient, we get

$$0 = A + C \qquad \left[\because C = \frac{1}{4}\right]$$

$$0 = A + \frac{1}{4}$$

$$-\frac{1}{4} = A$$

$$\therefore \boxed{A = -\frac{1}{4}}$$

Substitute A, B, C values in eq ① we get

$$\frac{1}{x^2(x+2)} = \frac{-1/4}{x} + \frac{1/2}{x^2} + \frac{1/4}{x+2}$$

$$= \frac{-1}{4x} + \frac{1}{2x^2} + \frac{1}{4(x+2)}$$

Resolve $\frac{3}{(x-1)(x+2)^2}$ into partial fractions. $\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$

Method - 3 (III)

Introduction

i. $\frac{x}{(x^2+2)(x+1)} = \frac{Ax+B}{(x^2+2)} + \frac{C}{(x+1)}$ 3rd Method

ii. $\frac{1}{(1-x^2)(x+2)} = \frac{1}{(1^2-x^2)(x+2)}$ $[a^2-b^2=(a+b)(a-b)]$

1st Method $= \frac{1}{(1+x)(1-x)(x+2)}$

$= \frac{A}{(1+x)} + \frac{B}{(1-x)} + \frac{C}{(x+2)}$

iii. $\frac{x+1}{(x^2-5x+6)(x+2)} = \frac{1}{(x-3)(x-2)(x+2)}$

1st Method $= \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x+2)}$

iv. $\frac{1}{(x^2+x+1)(x+3)} = \frac{Ax+B}{(x^2+x+1)} + \frac{C}{(x+3)}$ 3rd Method

(B) Resolve $\frac{x+2}{(1-x^2)(x^2+1)}$ into partial fractions

Sol $\frac{x+2}{(1-x^2)(x^2+1)} = \frac{x+2}{(1^2-x^2)(x^2+1)}$ $[a^2-b^2=(a+b)(a-b)]$

$= \frac{x+2}{(1-x)(1+x)(x^2+1)}$

Consider

$$\frac{x+2}{(1-x)(1+x)(x^2+1)} = \frac{A}{1-x} + \frac{B}{1+x} + \frac{Cx+D}{x^2+1} \longrightarrow (1)$$

$$\frac{x+2}{\cancel{(1-x)(1+x)(x^2+1)}} = \frac{A(1+x)(x^2+1)+B(1-x)(x^2+1)+(Cx+D)(1-x)(1+x)}{\cancel{(1-x)(1+x)(x^2+1)}}$$

$$x+2 = A(1+x)(x^2+1)+B(1-x)(x^2+1)+(Cx+D)(1-x)(1+x) \rightarrow (2)$$

To find A, B, C, D values

LCM

$$\Rightarrow \frac{(1-x)(1+x)(x^2+1)}{1-x}$$

$$\Rightarrow \frac{(1-x)(1+x)(x^2+1)}{1+x}$$

$$\Rightarrow \frac{(1-x)(1+x)(x^2+1)}{x^2+1}$$

Put $x=1$, we get

$1+2 = A(1+1)(1+1) + 0 + 0$

$3 = 4A$

$3/4 = A$

$\therefore \boxed{A = \frac{3}{4}}$

From eq (2) To find B value

Put $x=-1$, we get

$-1+2 = 0 + B(1-(-1))[(-1)^2+1] + 0$

$1 = 4B$

$1/4 = B$

$\boxed{B = \frac{1}{4}}$

From eq (2) to find C, D for simplify, we get

$x+2 = A[x^2+1+x^3+x] + B[x^2+1-x^3-x] + (Cx+D)(1^2-x^2)$

$x+2 = Ax^2+A+Ax^3+Ax+Bx^2+B-Bx^3-Bx+[Cx-Cx^3+D-Dx^2$

Now compare 'x^3' co-efficient, we get

$0 = A-B-C$

$0 = \frac{3}{4} - \frac{1}{4} - C$

$$0 = \frac{3-1}{4} - C$$

$$0 = \frac{\cancel{2}}{\cancel{4}\,2} - C$$

$$\cancel{-}\frac{1}{2} = \cancel{-}C$$

$$\therefore \boxed{C = \frac{1}{2}}$$

Now compare 'x^2' co-efficient, we get

$$0 = A + B - D$$

$$0 = \frac{3}{4} + \frac{1}{4} - D$$

$$0 = \frac{3+1}{4} - D$$

$$0 = \frac{\cancel{4}}{\cancel{4}} - D$$

$$0 = 1 - D$$

$$\cancel{-}1 = \cancel{-}D$$

$$\therefore \boxed{D = 1}$$

Substitute A, B, C, D values in eq ①, we get

$$\frac{x+2}{(1-x)(1+x)(x^2+1)} = \frac{3/4}{(1-x)} + \frac{1/4}{(1+x)} + \frac{\frac{1}{2}x+1}{(x^2+1)}$$

$$= \frac{3}{4(1-x)} + \frac{1}{4(1+x)} + \frac{\frac{1}{2}x+1}{x^2+1} \Rightarrow \frac{x+2}{2(x^2+1)}$$

(B) Resolve $\frac{2x+1}{(x-1)(x^2+1)}$ into partial fractions.

H.W **Method - I**

(i) Resolve $\frac{x-4}{x^2-5x+6}$

Sol $$\frac{x-4}{x^2-5x+6} = \frac{x-4}{x^2-3x-2x+6}$$

Hint
$1 \times 6 = 6$
$-3 \quad -2$

$$= \frac{x-4}{x(x-3)-2(x-3)}$$

(i) two number multiply $= 6$

(ii) two number add $= -5x$

$$\frac{x-4}{x^2-5x+6} = \frac{x-4}{(x-3)(x-2)} \longrightarrow (1)$$

<u>Consider</u>

$$\frac{x-4}{(x-3)(x-2)} = \frac{A}{(x-3)} + \frac{B}{(x-2)} \longrightarrow (2)$$

$$\frac{x-4}{\cancel{(x-3)(x-2)}} = \frac{A(x-2)+B(x-3)}{\cancel{(x-3)(x-2)}}$$

$$x-4 = A(x-2)+B(x-3) \longrightarrow (3)$$

To find A, B values

Find 'A'

Put $x=3$, we get

$$3-4 = A(3-2)+B(3-3)$$

$$-1 = A(1)+0$$

$$-1/1 = A$$

$$\therefore \boxed{A=-1}$$

From eq (3). To find 'B' value

Put $x=2$, we get

$$x-4 = A(x-2)+B(x-3)$$

$$2-4 = A(2-2)+B(2-3)$$

$$-2 = 0+B(-1)$$

$$\cancel{-}2 = \cancel{-}1B$$

$$2/1 = B$$

$$\therefore \boxed{B=2}$$

Substitute A, B values in eq (2), we get

$$\frac{x-4}{(x-3)(x-2)} = \frac{-1}{(x-3)} + \frac{2}{(x-2)}$$

H.W 11. Resolve $\frac{2x-1}{(x-1)(2x+3)}$

Sol

$$\frac{2x-1}{(x-1)(2x+3)} = \frac{A}{(x-1)} + \frac{B}{(2x+3)} \quad \text{--- (1)}$$

To find A, B values

$$\frac{2x-1}{\cancel{(x-1)(2x+3)}} = \frac{A(2x+3)+B(x-1)}{\cancel{(x-1)(2x+3)}}$$

$$2x-1 = A(2x+3)+B(x-1) \quad \text{--- (2)}$$

To find 'A' value

Put $x=1$, we get

$$2(1)-1 = A(2(1)+3)+B(1-1)$$
$$2-1 = A(2+3)+0$$
$$1 = 5A$$
$$1/5 = A$$
$$\therefore \boxed{A = \frac{1}{5}}$$

From eq (2) To find 'B' value

Put $x = -3/2$, we get

$2x+3=0$

$2x=-3$

$x = \frac{-3}{2}$

$$2x-1 = A(2x+3)+B(x-1)$$
$$2(-3/2)-1 = A(2(-3/2)+3)+B(-3/2-1)$$
$$\frac{-6-2}{2} = A\left(\frac{-6+6}{2}\right)+B\left(\frac{-3-2}{2}\right)$$
$$\frac{-8}{2} = 0 + B\left(\frac{-5}{2}\right)$$
$$\cancel{-}\frac{8}{\cancel{2}} = \cancel{-}\frac{5}{\cancel{2}}B$$
$$8/5 = B$$
$$\therefore \boxed{B = \frac{8}{5}}$$

Substitute A, B Values in equation ① we get

$$\frac{2x-1}{(x-1)(2x+3)} = \frac{1/5}{(x-1)} + \frac{8/5}{(2x+3)}$$

$$= \frac{1}{5(x-1)} + \frac{8}{5(2x+3)}$$

Ⓑ **Method - II**

H.W Resolve $\frac{3}{(x-1)(x+2)^2}$ into partial fractions

Sol

$$\frac{3}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2} \quad \text{—— ①}$$

$$\frac{3}{\cancel{(x-1)(x+2)^2}} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{\cancel{(x-1)(x+2)^2}}$$

$$\frac{\cancel{(x-1)}(x+2)^2}{\cancel{(x-1)}} = (x+2)^2$$

$$\frac{(x-1)(x+2)^2}{(x+2)} = (x-1)(x+2)$$

$$\frac{(x-1)\cancel{(x+2)^2}}{\cancel{(x+2)^2}} = (x-1)$$

$$3 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \quad \text{—— ②}$$

To find A, B, C values

Put $x = 1$, we get

$$3 = A(1+2)^2 + B(1-1)(1+2) + C(1-1)$$

$$3 = A((1)^2 + 2(1)(2) + (2)^2) + 0 + 0$$

$$3 = A(1+4+4) \qquad [(a+b)^2 = a^2 + 2ab + b^2]$$

$$3 = A(9)$$

$$\frac{\cancel{3}}{\cancel{9}_3} = A$$

$$1/3 = A$$

$$\therefore \boxed{A = 1/3}$$

From eq ② put $x = -2$, we get

$$3 = A(-2+2)^2 + B(-2-1)(-2+2) + C(-2-1)$$

$$3 = 0 + 0 + C(-3)$$

$$3 = -3C$$

$\cancel{3}/-\cancel{3} = C$

$-1 = C$

$\therefore \boxed{C = -1}$

From eq ② Simplify to find 'B' value, we get

$3 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

$3 = A(x^2+2(x)(2)+(2)^2) + B(x^2+2x-x-2) + Cx - C$

$(a+b)^2 = a^2+2ab+$

$a = x \ , b = 2$

$3 = Ax^2 + 4Ax + 4A + Bx^2 + Bx - 2B + Cx - C$

Now compare 'x^2' co-efficient, we get

$0 = A + B$ $\quad [\because A = \frac{1}{3}]$

$0 = \frac{1}{3} + B$

$-\frac{1}{3} = B$

$\therefore \boxed{B = \frac{-1}{3}}$

Substitute A, B, C values in eq ①, we get

$\frac{\cancel{2x+1}}{\cancel{(x+1)}} \quad \frac{3}{(x-1)(x+2)^2} = \frac{1/3}{(x-1)} + \frac{-1/3}{(x+2)} + \frac{-1}{(x+2)^2}$

$= \frac{1}{3(x-1)} + \frac{(-1)}{3(x+2)} + \frac{(-1)}{(x+2)^2}$

H.W Method - III

(B) Resolve $\frac{2x+1}{(x-1)(x^2+1)}$ into partial fractions.

Sol

$\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} \longrightarrow ①$

$\frac{2x+1}{\cancel{(x-1)(x^2+1)}} = \frac{A(x^2+1)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$

$2x+1 = A(x^2+1)+(Bx+C)(x-1) \longrightarrow ②$

Hint

$\frac{\cancel{(x-1)}(x^2+1)}{\cancel{(x-1)}} = (x^2+1)$

$\frac{(x-1)\cancel{(x^2+1)}}{\cancel{(x^2+1)}} = (x-1)$

To find A, B, C values

Put $x=1$, we get

$2(1)+1 = A((1)^2+1) + (B(1)+C)(1-1)$

$2+1 = A(1+1) + (B+C)(0)$

$3 = 2A + 0$

$$\frac{3}{2} = A$$

$$\therefore \boxed{A = \frac{3}{2}}$$

From eq ② To find B, C values for simplify, we get

$$2x+1 = A(x^2+1) + (Bx+C)(x-1)$$

$$2x+1 = Ax^2 + A + Bx^2 - Bx + Cx - C$$

Now compare 'x^2' co-efficient, we get

$$0 = A+B$$

$$0 = \frac{3}{2} + B$$

$$\frac{-3}{2} = B \qquad \therefore \boxed{B = -\frac{3}{2}}$$

Now compare 'x' co-efficient, we get

$$2 = -B + C$$

$$2 = -(-3/2) + C$$

$$2 = 3/2 + C$$

$$2 - \frac{3}{2} = C$$

$$\frac{4-3}{2} = C$$

$$\frac{1}{2} = C \qquad \therefore \boxed{C = \frac{1}{2}}$$

Substitute A, B, C values in eq ① we get

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3/2}{(x-1)} + \frac{-3/2\,x + 1/2}{(x^2+1)}$$

$$= \frac{3}{2(x-1)} + \frac{-3x+1}{2(x^2+1)} \;//$$

10/10/19

(8) Resolve $\frac{2x-3}{(x-1)(x^2+1)^2}$ into partial fractions

Sol

$$\frac{2x-3}{(x-1)(x^2+1)^2} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2} \quad \text{—— ①}$$

Find A, B, C, D, E Values

$$\frac{2x-3}{\cancel{(x-1)(x^2+1)^2}} = \frac{A(x^2+1)^2+(Bx+C)(x-1)(x^2+1)+(Dx+E)(x-1)}{\cancel{(x-1)(x^2+1)^2}}$$

$$2x-3 = A(x^2+1)^2+(Bx+C)(x-1)(x^2+1)+(Dx+E)(x-1) \quad \text{—— ②}$$

Put $x=1$ in eq ②, we get

$$2(1)-3 = A((1)^2+1)^2+0+0$$

$$2-3 = A(2)^2$$

$$-1 = 4A$$

$$\therefore \boxed{A = -1/4}$$

From eq ② simplify we have

$$2x-3 = A[(x^2)^2+2(x^2)(1)+(1)^2]+(Bx+C)[x^3+x-x^2-1]+Dx^2-Dx+Ex-E$$

$$2x-3 = Ax^4+2Ax^2+A+Bx^4+Bx^2-Bx^3-Bx+Cx^3+Cx-Cx^2-C+Dx^2-Dx+Ex-E \quad \text{—— ②}$$

Now compare 'x^4' coefficient, we have

$$0 = A+B$$

$$0 = -1/4+B$$

$$\therefore \boxed{B = \frac{1}{4}}$$

Now compare 'x^3' Coefficient, we have

$$0 = -B+C$$

$$0 = -1/4+C$$

$$\therefore \boxed{C = 1/4}$$

Now compare 'x^2' coefficient, we have

$$\cancel{0 = -C+D}$$

$$\cancel{0 = -1/4+D} \qquad \therefore \boxed{\cancel{D = 1/4}}$$

$0 = 2A + B - C + D$

$0 = 2\left(-\frac{1}{4}\right) + \frac{1}{4} - \frac{1}{4} + D$

$0 = -\frac{1}{2} + D$

$\therefore \boxed{D = \frac{1}{2}}$

Now compare 'x' coefficient, we have

$2 = -B + C - D + E$

$2 = -\frac{1}{4} + \frac{1}{4} - \frac{1}{2} + E$

$2 + \frac{1}{2} = E$

$\frac{4+1}{2} = E$

$\therefore \boxed{E = \frac{5}{2}}$

Substitute A, B, C, D and E values in eq ① we get

$$\frac{2x-3}{(x-1)(x^2+1)^2} = \frac{-1/4}{(x-1)} + \frac{\frac{1}{4}x + \frac{1}{4}}{(x^2+1)} + \frac{\frac{1}{2}x + \frac{5}{2}}{(x^2+1)^2}$$

$$= \frac{-1}{4(x-1)} + \frac{x+1}{4(x^2+1)} + \frac{x+5}{2(x^2+1)^2} \text{ //}$$

③ Resolve $\frac{x^2+1}{(x^2+x+1)^2}$ into partial fractions.

Sol $\frac{x^2+1}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$ —— ①

To find A, B, C and D values.

$$\frac{x^2+1}{\cancel{(x^2+x+1)^2}} = \frac{(Ax+B)(x^2+x+1)+(Cx+D)(1)}{\cancel{(x^2+x+1)^2}}$$

$x^2+1 = (Ax+B)(x^2+x+1) + (Cx+D)$ —— ②

Put From eq ②, simplify we have

$x^2+1 = Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$

Now, compare 'x^3' coefficient, we get

14/10/2019

Definitions

Polynomial :- An expressions

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$, where $a_i \in R, 0 \le i \le n$ is called a polynomial in x with real co-efficient.

Ex:- x^3+3x^2+2x+4, x^2+6x-5.

Degree of a Polynomial :- If $f(x) =$

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$ and $a_0 \neq 0$ then 'n' is called degree of $f(x)$.

Ex:- 1. The degree of polynomial x^3+2x+1 is 3.

2. $x^5+4x^3+2x^2+x+1$ is degree 5.

Rational Fraction :- If $f(x), g(x)$ are two polynomials and $g(x)$ is non zero polynomial then $\frac{f(x)}{g(x)}$ is called a rational fraction (or) simply a fraction.

Ex:- 1. $\frac{x^2+4x+1}{x+3}$ is a rational fraction.

2. $\frac{x+2}{x^2+3x+2}$ is a rational fraction.

Proper fraction :- A rational fraction $\frac{f(x)}{g(x)}$ is called a proper fraction, if the degree of $f(x)$ is less than the degree of $g(x)$.

Ex:- 1. $\frac{2x+1}{x^3+2}$ is a proper fraction.

2. $\dfrac{x^3}{x^4+3x^2+2}$

Improper Fraction :- A rational fractional $\dfrac{f(x)}{g(x)}$ an Improper fraction, if the degree of $f(x)$ is gr than (or) equal to degree of $g(x)$.

Ex:- 1. $\dfrac{x^4}{x^2+3x+5}$ is an improper fraction

2. $\dfrac{x^3}{x^3+2x^2-1}$ is an improper fraction.

Limits

Basic Formulas

1. If $\underset{x \to a}{Lt} \dfrac{x^n-a^n}{x-a} = na^{n-1}$

2. $\underset{x \to a}{Lt} \dfrac{x^m-a^m}{x^n-a^n} = \dfrac{m}{n} a^{m-n}$

3. $\underset{\theta \to 0}{Lt} \dfrac{\sin\theta}{\theta} = 1$

4. $\underset{\theta \to 0}{Lt} \dfrac{\tan\theta}{\theta} = 1$

Note :- $\dfrac{0}{0}, \infty, \dfrac{\infty}{\infty}, \dfrac{1}{0}$ limits does not exist.

Evalute $\lim_{x \to 2} \frac{x+5}{3x^2-4x+3}$.

Sol

$$\lim_{x \to 2} \frac{x+5}{3x^2-4x+3}$$

$$= \frac{(2)+5}{3(2)^2-4(2)+3} = \frac{7}{12-8+3}$$

$$\Rightarrow \frac{7}{15-8} = \frac{\cancel{7}}{\cancel{7}} = 1$$

Evaluate $\lim_{x \to 2} \frac{x^2-4}{x-2}$

Sol

$$\lim_{x \to 2} \frac{x^2-4}{x-2}$$

$$= \frac{(2)^2-4}{2-2} = \frac{4-4}{2-2} = \frac{0}{0}$$ Limit does not exist.

$$\lim_{x \to 2} \frac{x^2-4}{x-2}$$

$\left[\text{Formula:- } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$

$a=2, n=2$

$$\lim_{x \to 2} \frac{x^2-(2)^2}{x-2} = (2)(2)^{2-1}$$

$$= (2)(2)^1$$

$$= 4$$

Evaluate $\underset{x\to 2}{\mathrm{Lt}} \dfrac{x^4-16}{x^2-4}$

<u>Sol</u> $\underset{x\to 2}{\mathrm{Lt}} \dfrac{(2)^4-16}{(2)^2-4}$

$$= \frac{16-16}{4-4} = \frac{0}{0}$$

Limit does not exist.

$\underset{x\to 2}{\mathrm{Lt}} \dfrac{x^4-16}{x^2-4}$

$\left[\text{Formula. } \underset{x\to a}{\mathrm{Lt}} \dfrac{x^m-a^m}{x^n-a^n} = \dfrac{m}{n} a^{m-n}\right]$

$a=2,\ m=4,\ n=2$

$$\underset{x\to 2}{\mathrm{Lt}} \frac{x^4-2^4}{x^2-(2)^2} = \frac{\cancel{4}2}{\cancel{2}}(2)^{4-2}$$

$$= 2(2)^2$$

$$= 8 \,//$$

15-10-2019

Evaluate $\underset{x\to 2}{\mathrm{Lt}} \dfrac{\sqrt{x^3}-\sqrt{8}}{\sqrt{x^5}-\sqrt{32}}$

<u>Sol</u> $\underset{x\to 2}{\mathrm{Lt}} \dfrac{\sqrt{x^3}-\sqrt{8}}{\sqrt{x^5}-\sqrt{32}}$

$\underset{x\to 2}{\mathrm{Lt}} \dfrac{(x^3)^{1/2}-(8)^{1/2}}{(x^5)^{1/2}-(32)^{1/2}}$

$\underset{x\to 2}{\mathrm{Lt}} \dfrac{(x)^{3/2}-(8)^{1/2}}{(x)^{5/2}-(32)^{1/2}}$

$$\left[\lim_{x\to a}\frac{x^m-a^m}{x^n-a^n}=\frac{m}{n}a^{m-n}\right]$$

$a=2$, $m=\frac{3}{2}$, $n=\frac{5}{2}$

$$\lim_{x\to 2}\frac{x^{3/2}-(2)^{3/2}}{x^{5/2}-(2)^{5/2}}=\frac{3/\cancel{2}}{5/\cancel{2}}(2)^{3/2-5/2}$$

$$=\frac{3}{5}(2)^{\frac{3-5}{2}}$$

$$=\frac{3}{5}(2)^{\frac{-\cancel{2}}{\cancel{2}}}$$

$$=\frac{3}{5}(2)^{-1}$$

$$=\frac{3}{5}\cdot\frac{1}{(2)}$$

$$=\frac{3}{10}\ ||$$

Evaluate

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

Sol

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

$$\lim_{x\to 0}\frac{\overset{a}{\sqrt{1+x}}-\overset{b}{\sqrt{1-x}}}{2x}\times\frac{(\overset{a}{\sqrt{1+x}}+\overset{b}{\sqrt{1-x}})}{\sqrt{1+x}+\sqrt{1-x}}$$

$(a-b)(a+b)=a^2-b^2$

$a=\sqrt{1+x}$, $b=\sqrt{1-x}$

$$\lim_{x\to 0}\frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{2x[\sqrt{1+x}+\sqrt{1-x}]}$$

$$\lim_{x\to 0}\frac{\cancel{1}+x-\cancel{1}+x}{2x[\sqrt{1+x}+\sqrt{1-x}]}$$

$$\lim_{x\to 0}\frac{\cancel{2x}}{\cancel{2x}[\sqrt{1+x}+\sqrt{1-x}]}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$$

$$=\frac{1}{\sqrt{1+0}+\sqrt{1-0}}=\frac{1}{1+1}=\frac{1}{2}\ ||$$

Evaluate $\underset{x \to 0}{Lt} \frac{\sqrt{x+1}-1}{x}$

Evaluate $\underset{x \to \infty}{Lt} \frac{3x^2-5x+9}{2x^2+4x+7}$

Sol

$$\underset{x \to \infty}{Lt} \frac{3x^2-5x+9}{2x^2+4x+7}$$

$$= \frac{\infty}{\infty}$$

Limit does not exist

$$\underset{x \to \infty}{Lt} \frac{3x^2-5x+9}{2x^2+4x+7}$$

$$= \underset{x \to \infty}{Lt} \frac{\cancel{x^2}\left[3-\frac{5x}{x^2}+\frac{9}{x^2}\right]}{\cancel{x^2}\left[2+\frac{4x}{x^2}+\frac{7}{x^2}\right]}$$

$$= \underset{x \to \infty}{Lt} \frac{3-5/x+9/x^2}{2+4/x+7/x^2}$$

$$= \frac{3-5/\infty+9/\infty}{2+4/\infty+7/\infty} \quad [\because \infty^2 = \infty]$$

$$\left[\frac{1}{\infty}=0\right], \frac{2}{\infty}, \frac{3}{\infty}, \frac{4}{\infty}, \frac{5}{\infty}$$

$$= \frac{3-0+0}{2+0+0}$$

$$= \frac{3}{2} \,//$$

Evaluate

$$\underset{x \to 0}{Lt} \frac{\sin 2x}{\sin 3x}$$

Sol

$$\underset{x \to 0}{Lt} \frac{\sin 2x}{\sin 3x}$$

$$\left[\underset{\theta \to 0}{Lt} \frac{\sin \theta}{\theta} = 1\right]$$

$$= \frac{\lim\limits_{x\to 0} \sin 2x}{\lim\limits_{x\to 0} \sin 3x}$$

$$= \frac{\lim\limits_{x\to 0} \frac{\sin 2x}{2x} \times 2\cancel{x}}{\lim\limits_{x\to 0} \frac{\sin 3x}{3x} \times 3\cancel{x}}$$

$$= \frac{2}{3} \frac{\left(\lim\limits_{2x\to 0} \frac{\sin 2x}{2x}\right)}{\left(\lim\limits_{3x\to 0} \frac{\sin 3x}{3x}\right)}$$

$$= \frac{2}{3} \frac{(1)}{(1)}$$

$$= \frac{2}{3} \;//$$

Evaluate $\lim\limits_{x\to 0} \frac{\tan 3x}{3}$

Sol

$$\lim\limits_{x\to 0} \frac{\tan 3x}{3}$$

$$\left[\lim\limits_{\theta\to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= \lim\limits_{x\to 0} \frac{\tan 3x}{3x} \times x$$

$$= \left[\lim\limits_{x\to 0} \frac{\tan 3x}{3x}\right] \times \left[\lim\limits_{x\to 0} x\right]$$

$$= \left[\lim\limits_{3x\to 0} \frac{\tan 3x}{3x}\right] \times 0$$

$$= 0 \;//$$

H.W

Evaluate $\lim\limits_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

Sol

$$\lim\limits_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

$$= \frac{\sqrt{0+1}-1}{0}$$

$$= \frac{0}{0}$$ limit does not exist

$\therefore$ consecutive method

$$\lim\limits_{x\to 0} \frac{(\sqrt{x+1}-1)}{x} \times \frac{(\sqrt{x+1}+1)}{\sqrt{x+1}+1}$$

$$\lim\limits_{x\to 0} \frac{(\sqrt{x+1})^2-(1)^2}{x(\sqrt{x+1}+1)}$$

$(a-b)(a+b) = a^2-b^2$

$$\lim\limits_{x\to 0} \frac{x+\cancel{1}-\cancel{1}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$\lim\limits_{x\to 0} \frac{1}{\sqrt{x+1}+1}$$

$$= \frac{1}{\sqrt{0+1}+1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2} \;//$$

Logarithm:

Basic Formula :-

1. $\log_a a = 1$

Ex:- $\log_3 3 = 1$

2. $\log_a 1 = 0$

3. $b \log_e a = \log_e a^b$

4. $\log_e a + \log_e b = \log_e (ab)$

5. $\log_e a - \log_e b = \log_e (a/b)$

Example - 1 Prove that $\log_{10} 1600 = 2 + 4 \log_{10} 2$

Sol To prove that $\log_{10} 1600 = 2 + 4 \log_{10} 2$

$RHS = 2 + 4 \log_{10} 2$

$= (2)(1) + \log_{10} 2^4$ $\quad [b \log a = \log a^b]$

$= (2) \log_{10} 10 + \log_{10} 16$

$= \log_{10} 10^2 + \log_{10} 16$ $\quad [\log_e a + \log_e b = \log_e (ab)]$

$= \log_{10} 100 + \log_{10} 16$

$= \log_{10} 1600$

$L.H.S = R.H.S$

16/10/2019

H.W Prove that $\log 0.00125 = 3 - 5 \log_5 10$

Find the value of $\log_3 (1 + 1/3) + \log_3 (1 + 1/4) + \cdots + \log_3 (1 + \frac{1}{80}$

Sol $\log_3 (1 + 1/3) + \log_3 (1 + 1/4) + \cdots\cdots + \log_3 (1 + 1/80)$

$= \log_3 (4/3) + \log_3 (5/4) + \cdots\cdots + \log_3 \left(\frac{81}{80}\right)$ $\left[\log a/b = \log a - \log b\right]$

$= \cancel{\log_3 4} - \log_3 3 + \cancel{\log_3 5} - \cancel{\log_3 4} \cdots\cdots + \log_3 81 - \cancel{\log_3 80}$

$\log_3 (1+1/5) = \log_3 \frac{6}{5} = \log_3 6 - \log_3 5$

$= -\log_3 3 + \log_3 81$

$= -1 + \log_3 (3)^4$ $\left[\log_e a^b = b \log a\right]$

$= -1 + 4 \log_3 3$

$= -1 + 4(1)$

$= 3$ //

H.W Prove that $\log 0.00125 = 3 - 5 \log_5 10$

Sol To prove that $\log 0.00125 = 3 - 5 \log_5 10$

R.H.S $= 3 - 5 \log_5 10$ $\left[b \log a = \log a^b\right]$

$= 3(1) - \log_5 10^5$

$= 3 \log_5 5 - \log_5 10^5$

$= \log_5 5^3 - \log_5 10^5$ $\left[\log_e a - \log_e b = \log_e (a/b)\right]$

$= \log_5 125 - \log_5 100000$

$= \log_5 \frac{125}{100000}$

$= \log 0.00125$

R.H.S = L.H.S